KINEMATIC ANALYSIS
OF
CONSTANT–BREADTH CAM–FOLLOWER MECHANISMS

By

N. RAJARAM

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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By

N. RAJARAM

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1981

Dedicated to

MY PARENTS

CERTIFICATE

This is to certify that the work entitled, "Kinematic Analysis of Constant-Breadth Cam-Follower Mechanisms" by N.Rajaram has been carried out under my supervision and has not been submitted elsewhere for a degree.

Dr.S.G.Dhande,
Assistant Professor,
Department of Mechanical Engineering,
Indian Institute of Technology,
Kanpur.

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N.Rajaram

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NOMENCLATURE

$[L_{\bar{2}2}]$, $[L_{\bar{2}1}]$, $[M_{\bar{2}2}]$, $[M_{\bar{2}1}]$	Geometrical transformation Matrices
P, Q	Points of contact of the cam and the follower
$\underset{\sim}{n}_2^{(P)}$, $\underset{\sim}{n}_1^{(P)}$	Unit normal vector of the cam and the follower
$\underset{\sim}{R}_2^{(P)}$, $\underset{\sim}{R}_1^{(P)}$	Radius vector of the cam and the follower
s_1	The linear displacement of the follower
s_1'	The linear velocity ratio of the follower
s_1''	The linear acceleration ratio of the follower
$S(XOY)$	Global coordinate system
$S_2(X_2O_2Y_2)$, $S_1(X_1O_1Y_1)$	Moving coordinate systems
$S_{\bar{2}}(X_{\bar{2}}O_{\bar{2}}Y_{\bar{2}})$, $S_{\bar{1}}(X_{\bar{1}}O_{\bar{1}}Y_{\bar{1}})$	Initial coordinate systems
δ	Vertical linear displacement parameter
η	Horizontal linear displacement parameter
θ	The angle between the point of contact on the cam profile and the X axis
ϕ_2	Parameter of the angle of motion of the cam
ϕ_1	Parameter of the angle of rotation of the follo
ϕ_1'	Angular velocity ratio of the follower
ϕ_1''	Angular acceleration ratio of the follower
ω_2	Angular velocity of the cam

ABSTRACT

In this work, kinematic analysis of a special class of cam-follower mechanisms having constant-breadth cam profiles has been carried out. The objective of this kinematic analysis is to evaluate the displacement, the velocity and the acceleration of the follower driven by a constant-breadth cam profile of the type chosen. The two different cam profiles, which have been selected, can be described as follo

(i) The first cam profile is formed by three consecutive pairs of circular arcs obtained by taking three vertices of a triangle as centres and keeping the sum of the radii of the opposite circular arcs in a pair constant. This profile is named as Type I cam profile.

(ii) The second cam profile is the locus of the endpoints of a constant length rod when it rolls over a three cusped hypocycloid. This profile is named as Type II cam profile.

Using a flat-faced follower, expressions for displacemen velocity and acceleration have been derived for the following cases:

(a) Type I Cam Profile and

- Translating, On-centre Follower,
- Translating, Offset Follower, and
- General Case (Translating as well as Oscillating Follower).

(b) Type II Cam Profile and

- Translating, On-centre Follower,
- Oscillating Follower, and
- General Case.

The theoretical work has been followed by developing appropriate computer programs. Numerical examples have been worked out for each mechanism mentioned above. Results of the computational work have been presented and discussed.

Chapter 1

INTRODUCTION

1.1 Profile-Closed Higher Pair Mechanisms

In general, higher pair mechanisms can be classified into two categories, namely, externally closed mechanisms and profile closed mechanisms[1]. For externally closed higher pair mechanisms a restraining force is needed to keep the follower in contact with the cam-profile. Such a restraining force is generally provided by a spring attached to the follower. For instance, in an internal combustion engine, the valves need to close firmly as well as the contact between the cam and the follower profile need to be maintained throughout the working cycle. This is accomplished by providing a compression spring on the valve rod. Though necessary for maintaining contact, it is not always essential from kinematic viewpoint to have a spring loaded follower. For example in the case of profile closed mechanisms, which are largely used in automatic machinery, the follower can be in the form of a groove of constant width formed between two parallel profiles. The cam profile is contacting on either side of such a follower giving a positive constraint for relative motion between the cam and the follower links.

In gear mechanisms, maintaining contact between a pair of profile poses a problem. However, in this case, the inertial loading on gears as well as provision of gear teeth on either side prevent any undesirable separation of contacting profiles.

Moreover, if backlash is reduced to zero, then gears can also be considered as profile-closed higher pair mechanisms[1,2,3].

As mentioned by Hunt[1] - "There are other forms of profile-closed higher pairs ..... (in which) ..... the two engaging elements of such a pair are themselves inherently profile-closed without their requiring added profiles to effect the desired constraint. Such pairs have practical utility as well as being interesting from the geometrical point of view".

1.2 Previous Work - a Survey

Since the kinematics of constant-breadth cam-follower mechanisms depends on the geometry of the contacting surfaces, many investigators have made use of different geometrical constructions to obtain constant-breadth cam profiles. Notable among them are Hunt[1], Reuleaux[2], Rothbart[4], Jenson[5], Wunderlich[6] and Goldberg[7].

Reuleaux[2] has described in detail different types of profile-closed higher pair mechanisms. The profiles used are of various types; in each case the other contacting profile has been selected on the basis that this profile is fixed in space. The fixed profiles are a triangle, a square, a pentagon etc. In each case Reuleaux[2] has carried out extensive geometrical analysis to derive the centrodes of relative motion between a pair of profile-closed higher pair elements. Though elegant, the work is not amenable for synthesis purposes.

Rothbart[4] has presented a constant-breadth cam profile. This profile is formed by three circular arcs drawn by taking the vertices of an equilateral triangle as centres and the lengths of all sides of this triangle as radii. The analysis procedure is graphical in nature.

Jenson[5] has analysed a constant-breadth cam profile mechanism. The profile formed is such that part of it is formed by drawing two opposite circular arcs with a common centre, and radii as r_1 and r_2 where r_1 is greater than r_2. These arcs are joined by circular arcs each of them having a radius (r_1+r_2) and their centres at the corners of the circular arc of radius r_1.

Wunderlich[6] has analysed a constant-breadth cam-follower mechanism. The follower is in the form of a square yoke, oscillating around a fixed pivot and driven by a three cusped hypocycloid. Wunderlich[6] concludes that the mechanism works like a pair of gears having a constant velocity ratio.

Goldberg[7] has proposed a number of different types of mechanisms formed by using constant-breadth cam profile. Though a large number of mechanisms have been proposed kinematic analysis for each one of them has not been provided.

1.3 Present Work - an Outline

The objective of the present work is to develop a unified procedure of kinematic analysis for different types of

constant-breadth cam-follower mechanisms. General theories of kinematic analysis of higher pair mechanisms have been proposed by Litvin[8] and Dyson[9]. Dhande and Chakraborty[10] have used the general theories to develop procedure of kinematic analysis and synthesis of cam mechanisms commonly encountered in practice. Following a similar approach, cam profiles having constant-breadth have been investigated in the present work.

A constant-breadth cam profile consists of several different curves joined together at their ends to form a closed profile. Because of such composite nature it is difficult to describe such curves by one parametric equation. The difficulty is accentuated when it is necessary to carry out the kinematic analysis since one has to locate that section of the profile in which the point of contact lies. It is for these reasons that extensive analytical work has not been reported in the literature of constant-breadth cam-follower mechanisms.

Several constant-breadth cam-follower mechanisms formed by using two types of profiles have been analysed in the present work. In Chapter 2, kinematic analysis of cam-follower mechanisms using the Type I constant-breadth cam profile have been analysed. In the two cases discussed in Chapter 2, the follower is having translatory motion and is having a flat-faced profile. Numerical examples have been presented to illustrate the procedure developed.

In Chapter 3, kinematic analysis of constant-breadth cam-follower mechanisms having oscillating and translating flat-faced followers are given. The cam profile in this case is of Type II. In this chapter also appropriate numerical examples are given.

In Chapter 4, follower is considered to have two degrees of freedom - translating as well as oscillating through a slotted pivot. The follower profile consists of a pair of right-angled flat-faces. The follower contacts a constant-breadth cam profile at two different points simultaneously. Such mechanisms produce a general planar motion of the follower link. Two mechanisms of such type using Type I and Type II cam profiles are studied in the present work. Results of analytical and computational work are presented in Chapter 4.

Chapter 5 summarizes the work done. Suitable suggestions for further work in this area are also given in this chapter.

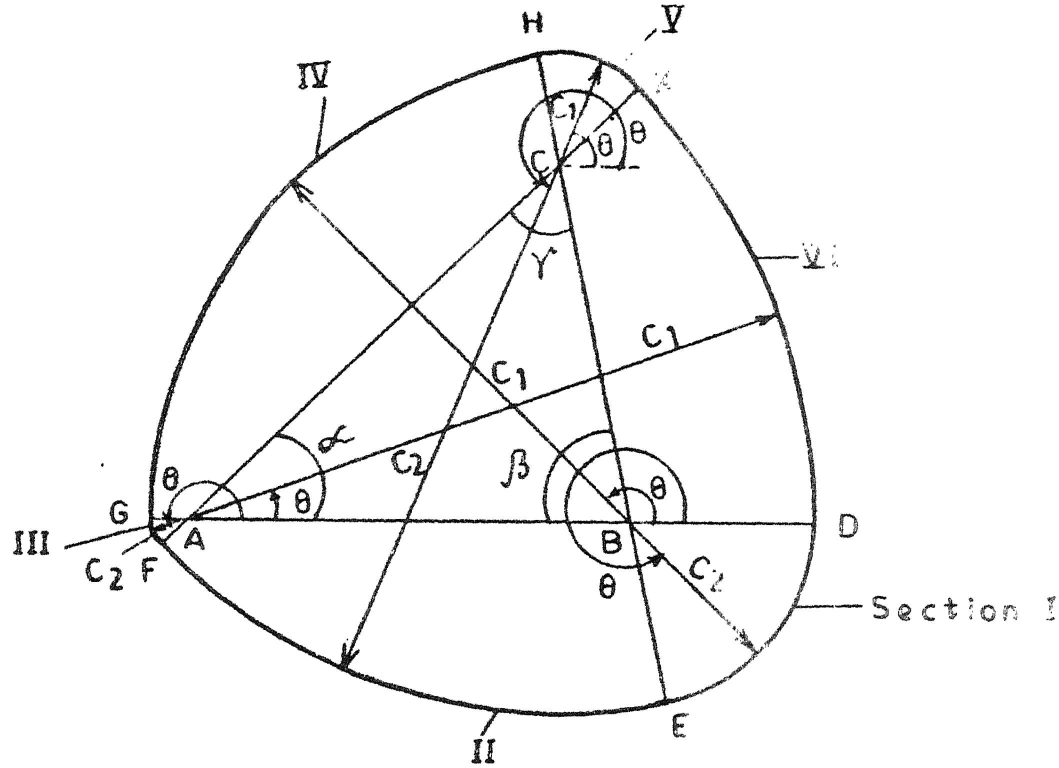
Chapter 2

KINEMATIC ANALYSIS OF CONSTANT-BREADTH CAM-FOLLOWER MECHANISMS - TYPE I

2.1 Type I Cam Profile

One important type of a profile used for constant-breadth cam mechanisms is shown in Figure 2.1. The profile consists of three pairs of circular arcs formed by taking vertices of a triangle ABC as centres. When a pair of arcs is formed with one vertex as the centre, then the sum of the radii of this pair of arcs is kept constant. This is being referred to as Type I constant-breadth cam profile. Figure 2.2 a shows a Type I profile being used as a constant-breadth cam rotating between parallel flat follower-surfaces attached to a sliding output-member which, in turn, is reciprocating between dwells at its ends of travel.

Consider a Type I profile shown in Figure 2.1 as described in a Cartesian coordinate system $X_2O_2Y_2$. The origin O_2 is located at the vertex B of the triangle ABC and the axis X_2 is along the side AB. The profile is divided into six sections in such a way that sections I and IV are formed by circular arcs with vertex B as the centre. Similarly sections II and V, III and VI are formed by the circular arcs with vertices A and C as the centres respectively. In order to describe the equation of the profile, it is necessary to determine as to which section a generic point P belongs.



AB = a = 6 . BC = b = 5 CA = c = 7 BD = d = 2.5

$C_1 + C_2 =$ CONSTANT

FIGURE 2.1

Type I Cam profile

Accordingly the parametric equation relating the coordinates x_2, y_2 to the parameter θ will vary. Let AB=a, BC=b and AC=c denote the sides of the triangle and BD=d such that $d > c-b$[1]. Table 2.1 describes the relation of coordinates (x_2, y_2) as functions of θ. It is important to note the bounds on the value of θ for each section.

Let $\underset{\sim}{n}_2^{(P)} = (n_{2x}, n_{2y})$ be a unit normal vector at the generic point P. Table 2.2 gives the expressions of n_{2x} and n_{2y} for all the six sections of the profile.

2.2 Kinematic Analysis of Translating, On-centre Follower

Consider a constant-breadth cam mechanism of Type I with a translating follower as shown in Figure 2.2a. The profile of the cam is described in Section 2.1. The cam rotates with a uniform angular velocity ω_2, with an angular rotation ϕ_2 as the parameter of the cam motion. The cam is rotating with vertex B as the pivot and at an initial position, side AB is along the direction of translation of the follower. Moreover, the linear displacement of the follower is measured from its initial position when the contact is maintained at point D. Coordinate systems used in the present analysis are shown in Figure 2.2b. $S_2(X_2O_2Y_2)$ and $S_1(X_1O_1Y_1)$ are the moving coordinate systems attached to the cam and the follower respectively. $S_{\bar{2}}(X_{\bar{2}}O_{\bar{2}}Y_{\bar{2}})$ and $S_{\bar{1}}(X_{\bar{1}}O_{\bar{1}}Y_{\bar{1}})$ are initial positions of the moving coordinate systems. $S(XOY)$ is the global coordinate system. The required transformation matrices for

Section	Profile	x_2	y_2	Range of θ
I	DE	$-d\cos\theta$	$-d\sin\theta$	$2\pi-\beta\leq\theta\leq2\pi$
II	EF	$-b\cos\beta-(b+d)\cos\theta$	$b\sin\beta-(b+d)\sin\theta$	$\pi+\alpha\leq\theta\leq2\pi-\beta$
III	FG	$-a-f\cos\theta$	$-f\sin\theta$	$\pi\leq\theta\leq\pi+\alpha$
IV	GH	$(a+f)\cos\theta$	$(a+f)\sin\theta$	$\pi-\beta\leq\theta\leq\pi$
V	HK	$-b\cos\beta+e\cos\theta$	$b\sin\beta+e\sin\theta$	$\alpha\leq\theta\leq\pi-\beta$
VI	KD	$(a+d)\cos\theta-a$	$(a+d)\sin\theta$	$0\leq\theta\leq\alpha$

Table 2.1 The parametric equations of Type I constant-breadth cam profile.

Section	I	II	III	IV	V	VI
n_{2x}	$-\cos\theta$	$-\cos\theta$	$-\cos\theta$	$\cos\theta$	$\cos\theta$	$\cos\theta$
n_{2y}	$-\sin\theta$	$-\sin\theta$	$-\sin\theta$	$\sin\theta$	$\sin\theta$	$\sin\theta$

Table 2.2 The x_2 and y_2 coordinates of unit normal vector

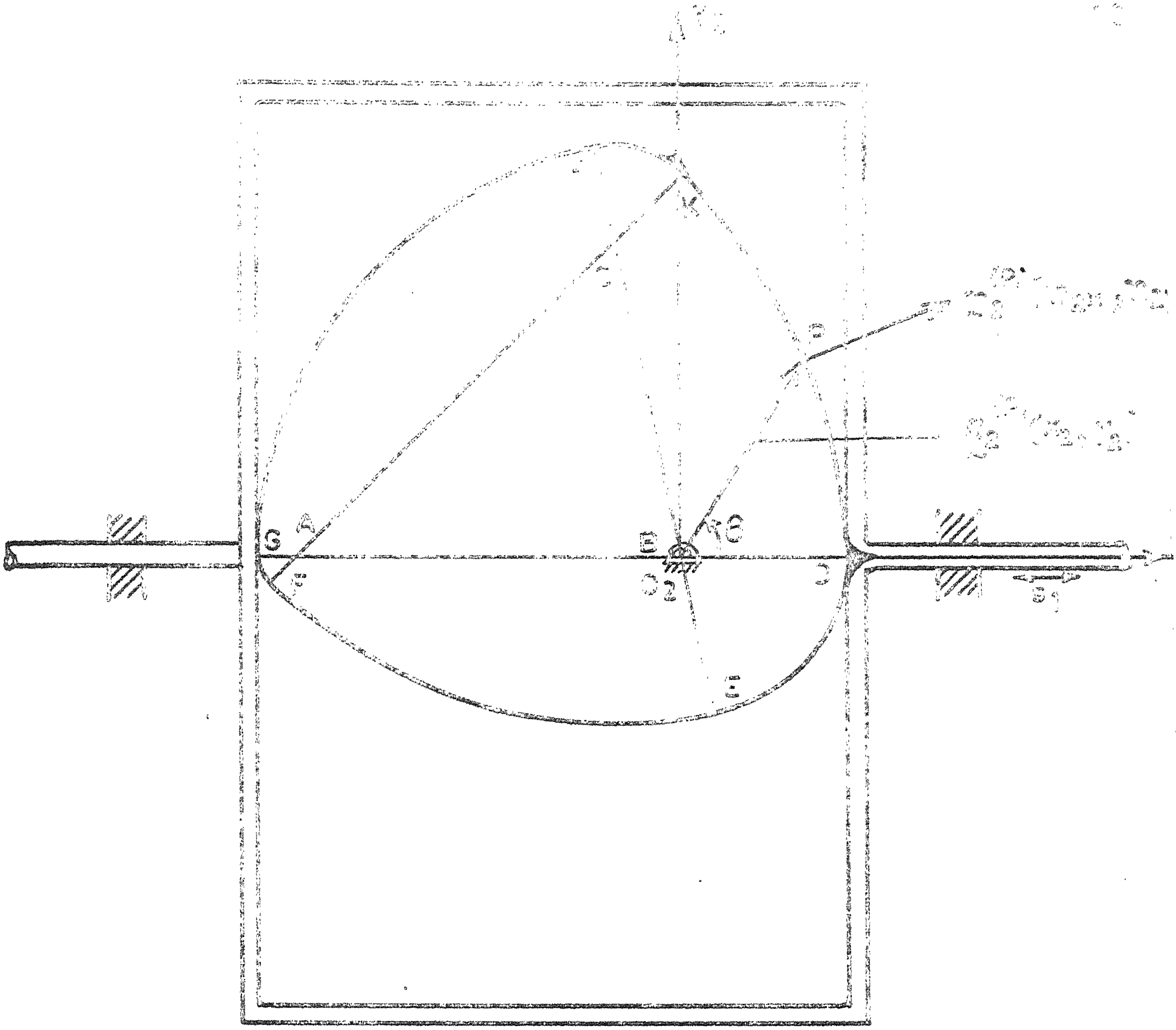


FIGURE 2.2 a

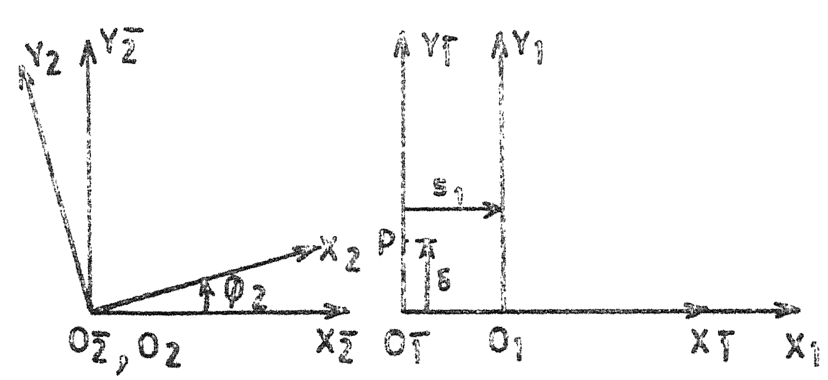


FIGURE 2.2 b

Type I Cam profile with ...

transforming radius vectors and unit normal vectors are given in Appendix I.

The objective of the kinematic analysis is to derive the expressions for displacement, velocity and acceleration of the follower, when the cam is rotating with a given uniform angular velocity, assuming that the parametric equations of the cam and the follower profiles are known. The relations of the displacement, velocity and acceleration of the follower will be parametric equations, the parameter being the angular position of the cam, ϕ_2.

Let $\underset{\sim}{R}_1^{(P)}$ and $\underset{\sim}{R}_2^{(P)}$ be the radius vectors of the cam and the follower surfaces respectively at a point P which is a point of contact between the cam and the follower profiles at an instant when the cam has been rotated through an angle ϕ_2. If the profiles are 'smooth' then following conditions can be written[10],

$$
\begin{aligned}
[M_{\bar{2}2}]\underset{\sim}{R}_2^{(P)} &= [M_{\bar{2}1}]\underset{\sim}{R}_1^{(P)} \\
[L_{\bar{2}2}]\underset{\sim}{n}_2^{(P)} &= [L_{\bar{2}1}]\underset{\sim}{n}_1^{(P)}
\end{aligned}
\qquad (2.1)
$$

where

$$
\begin{aligned}
\underset{\sim}{R}_1^{(P)} &= [0, \delta, 1]^T, \\
\underset{\sim}{R}_2^{(P)} &= [x_2, y_2, 1]^T, \\
\underset{\sim}{n}_1^{(P)} &= [1, 0]^T, \\
\underset{\sim}{n}_2^{(P)} &= [n_{2x}, n_{2y}]^T,
\end{aligned}
$$

distance δ locates point P on the follower surface as shown in Figure 2.2 and the transformation matrices are given in Appendix I.

Equations (2.1) form a set of three independent scalar equations. For a given value of ϕ_2, the unknowns are δ, s_1 and θ. Equations (2.1) can also be written as follows.

$$
\begin{aligned}
d + s_1 &= x_2\cos\phi_2 - y_2\sin\phi_2 \\
\delta &= x_2\sin\phi_2 + y_2\cos\phi_2 \qquad (2.2) \\
\theta + \phi_2 &= 2\pi
\end{aligned}
$$

Solving the above set of equations for θ, δ and s_1

$$\theta = 2\pi - \phi_2 \qquad (2.3a)$$

$$\delta = x_2\sin\phi_2 + y_2\cos\phi_2 \qquad (2.3b)$$

$$s_1 = x_2\cos\phi_2 - y_2\sin\phi_2 - d \qquad (2.3c)$$

where x_2, y_2 will be appropriate expressions from Table 2.1.

Equation (2.3a), for a given value of ϕ_2 indicates the value of angle θ. From Table 2.1, it is necessary to search for that section of the profile the range of which entraps this value of θ. Let j be the section that contacts the follower at any given instant. Since θ and ϕ_2 are known, x_2, y_2 for each section can be evaluated. Then using Equation (2.3c) the displacement s_1 can also be evaluated. The first and second derivatives of the displacement s_1 can be termed as the velocity ratio and the acceleration ratio respectively. The actual velocity and acceleration are

given by

$$\dot{s}_1 = \omega_2 s_1'$$
$$\ddot{s}_1 = \omega_2^2 s_1'' \qquad (2.4)$$

where

$$s_1' = \frac{ds_1}{d\phi_2}\text{, the velocity ratio,}$$
$$s_1'' = \frac{d^2 s_1}{d\phi_2^2}\text{, the acceleration ratio}$$

and ω_2, the angular velocity of the cam which is assumed to be constant.

For the present case expressions of s_1, s_1' and s_1'' are given in Table 2.3. It should be noted that all these expressions have been obtained after substituting the expressions of x_2 and y_2 from Table 2.1 and expression of θ from Equation (2.3a) in Equation (2.3c). This concludes the kinematic analysis procedure.

2.3 Kinematic Analysis of Translating, Offset Follower

Figure 2.3a illustrates a schematic diagram of an offset translating cam-follower system and Figure 2.3b shows the coordinate systems. The cam profile is still the same as described in Section 2.1, except that the rotating pivot of the cam is chosen 'ℓ' and 'm' distances from B in the horizontal and vertical directions. As explained in Section 2.1, the parametric equations for the coordinates of generic point P lying on the cam profile are determined for

Section	Displacement s_1	Velocity ratio s_1'	Acceleration ratio s_1''
I	0	0	0
II	$b(1 - \cos(\phi_2-\beta))$	$b \sin(\phi_2-\beta)$	$b \cos(\phi_2-\beta)$
III	$f - d - a \cos\phi_2$	$a \sin\phi_2$	$a \cos\phi_2$
IV	$a + b - c$	0	0
V	$a - c - b \cos(\phi_2-\beta)$	$b \sin(\phi_2-\beta)$	$b \cos(\phi_2-\beta)$
VI	$a(1 - \cos\phi_2)$	$a \sin\phi_2$	$a \cos\phi_2$

Table 2.3 The parametric equations of the displacement, the velocity ratio and the acceleration ratio of the translating, on-centre follower.

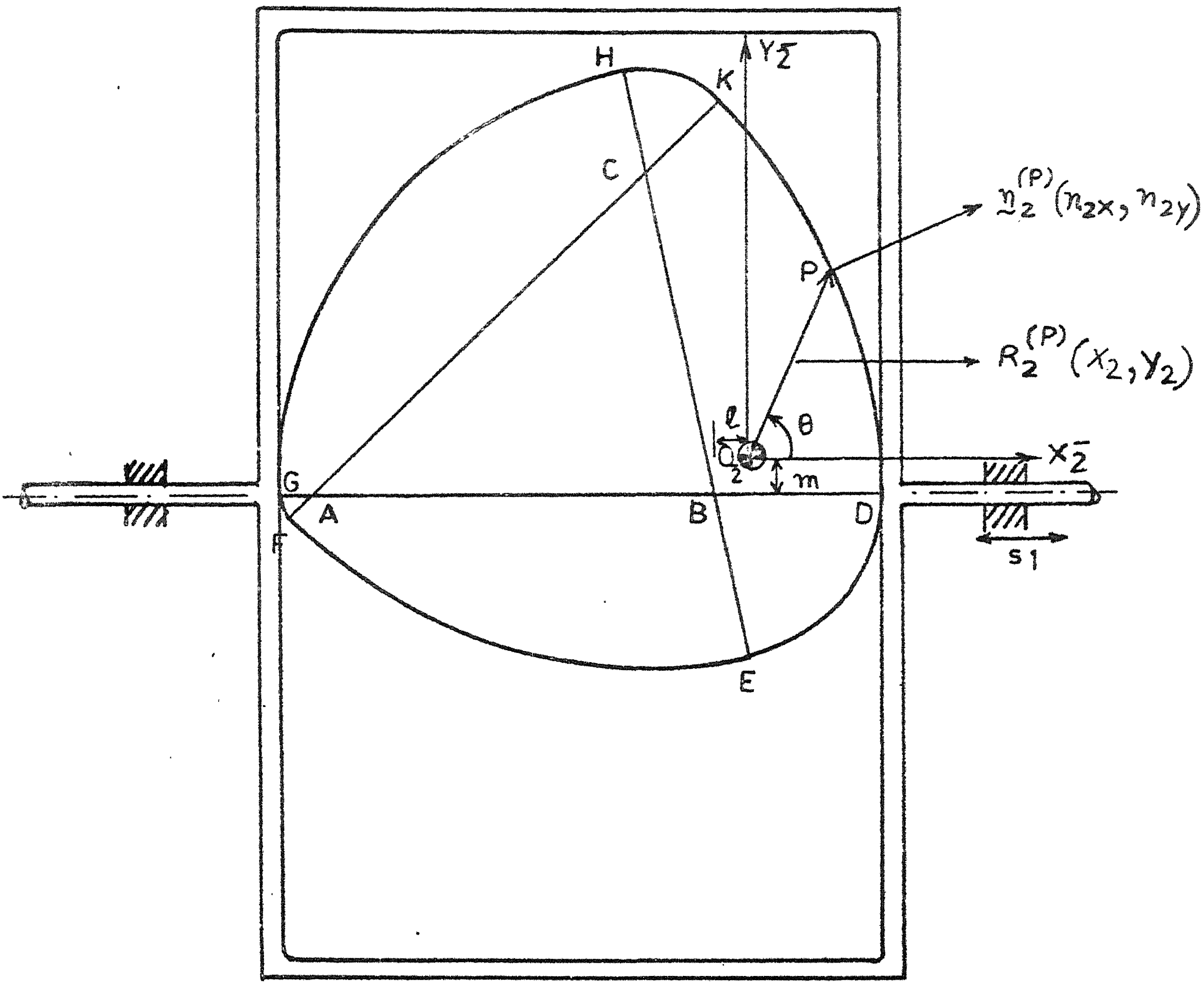


FIGURE 2.3 a

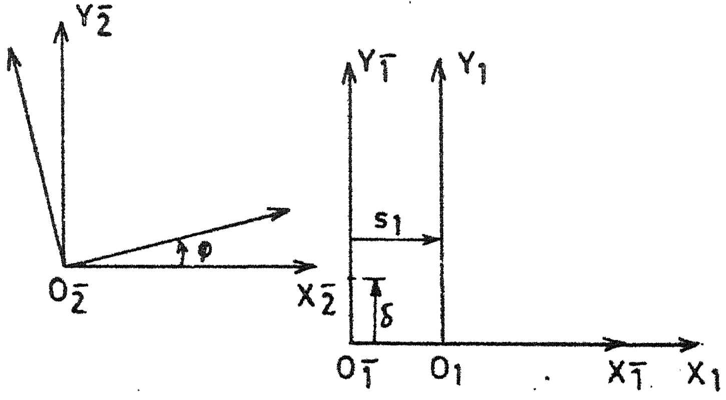


FIGURE 2.3 b

Type I Cam profile with offset translating follower.

all sections. The expressions for x_2 and y_2 are given in Table 2.4. Values for n_{2x} and n_{2y} will still be the same as Table 2.2.

Following the same procedure of kinematic analysis given in Section 2.2, expressions for the displacement, the velocity ratio and the acceleration ratio have been derived and presented in Table 2.5.

From the expression of the displacement, the velocity ratio, and the acceleration ratio derived for the present case, it can be seen that the corresponding equations for the case of an on-centre, translating follower can be derived by substituting $\ell = m = 0$.

2.4 Numerical Examples

Example #1

In order to illustrate the procedure of kinematic analysis presented in Section 2.2, for the on-centre translating follower, a cam profile with following data was selected.
a = 6 units, b = 5 units, c = 7 units and d = 2.5 units.
The profile generated is shown in Figure 2.1. A computer programme was developed to compute the displacement, velocity and acceleration of the follower. The uniform angular velocity is assumed to be unity. The step size for angle ϕ_2 was taken to be one degree. The curves showing variations of the displacement, velocity and acceleration of the follower as a function of ϕ_2, the angle of rotation, are shown in Figure 2.4.

ection	Profile	x_2	y_2	Range
I	DE	$-d\cos\theta-\ell$	$-d\sin\theta-m$	$2\pi-\beta\leq\theta\leq 2\pi$
II	EF	$-b\cos\beta-(b+d)\cos\theta-\ell$	$b\sin\beta-(b+d)\sin\theta-m$	$\pi+\alpha\leq\theta\leq 2\pi-\beta$
III	FG	$-f\cos\theta-a-\ell$	$-f\sin\theta-m$	$\pi\leq\theta\leq\pi-\alpha$
IV	GH	$(a+f)\cos\theta-\ell$	$(a+f)\sin\theta-m$	$\pi-\beta\leq\theta\leq\pi$
V	HK	$-b\cos\beta+e\cos\theta-\ell$	$b\sin\beta+e\sin\theta-m$	$\alpha\leq\theta\leq\pi-\beta$
VI	KD	$(a+d)\cos\theta-a-\ell$	$(a+d)\sin\theta-m$	$0\leq\theta\leq\alpha$

ble 2.4 The parametric equations of the offset translating constant-breadth cam profile.

Section	Displacement	Velocity ratio	Acceleration ratio
I	$\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$\ell\sin\phi_2 + m\cos\phi_2$	$\ell\cos\phi_2 - m\sin\phi_2$
II	$b(1-\cos(\phi_2-\beta))$ $+\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$b\sin(\phi_2-\beta)$ $+ \ell\sin\phi_2 + m\cos\phi_2$	$b\cos(\phi_2-\beta)$ $+ \ell\cos\phi_2$ $- m\sin\phi_2$
III	$(b-c) - a\cos\phi_2$ $+\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$a\sin\phi_2 + \ell\sin\phi_2$ $+ m\cos\phi_2$	$a\cos\phi_2 + \ell\cos\phi_2$ $- m\sin\phi_2$
IV	$(a+b+c)$ $+\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$\ell\sin\phi_2 + m\cos\phi_2$	$\ell\cos\phi_2 - m\sin\phi_2$
V	$(a-c) - b\cos(\phi_2-\beta)$ $+\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$b\sin(\phi_2-\beta)$ $+ \ell\sin\phi_2$ $+ m\cos\phi_2$	$b\cos(\phi_2-\beta)$ $+ \ell\cos\phi_2$ $- m\sin\phi_2$
VI	$a(1-\cos\phi_2)$ $+\ell(1-\cos\phi_2)$ $+ m\sin\phi_2$	$a\sin\phi_2$ $+ \ell\sin\phi_2$ $+ m\cos\phi_2$	$a\cos\phi_2$ $+ \ell\cos\phi_2$ $- m\sin\phi_2$

Table 2.5 The parametric equations for displacement, the velocity ratio, the acceleration ratio for Type I offset translating constant-breadth cam-follower mechanism.

Example # 2

In order to illustrate the procedure of kinematic analysis presented in Section 2.3, for the offset translating follower, a cam profile with following data was selected. $a = 6$ units, $b = 5$ units, $c = 7$ units, $d = 2.5$ units, $\ell = 0.5$ units and $m = 0.5$ units, where ℓ and m are the offset distances from B in X and Y directions.

The profile generated is shown in Figure 2.3a. A computer programme was developed to compute the displacement, velocity and acceleration of the follower. The uniform angular velocity is assumed to be unity. The step size of the angle ϕ_2 is taken to be one degree. The curves showing variations of the displacement, velocity and acceleration of the follower as a function of ϕ_2, the angle of rotation, are shown in Figure 2.5.

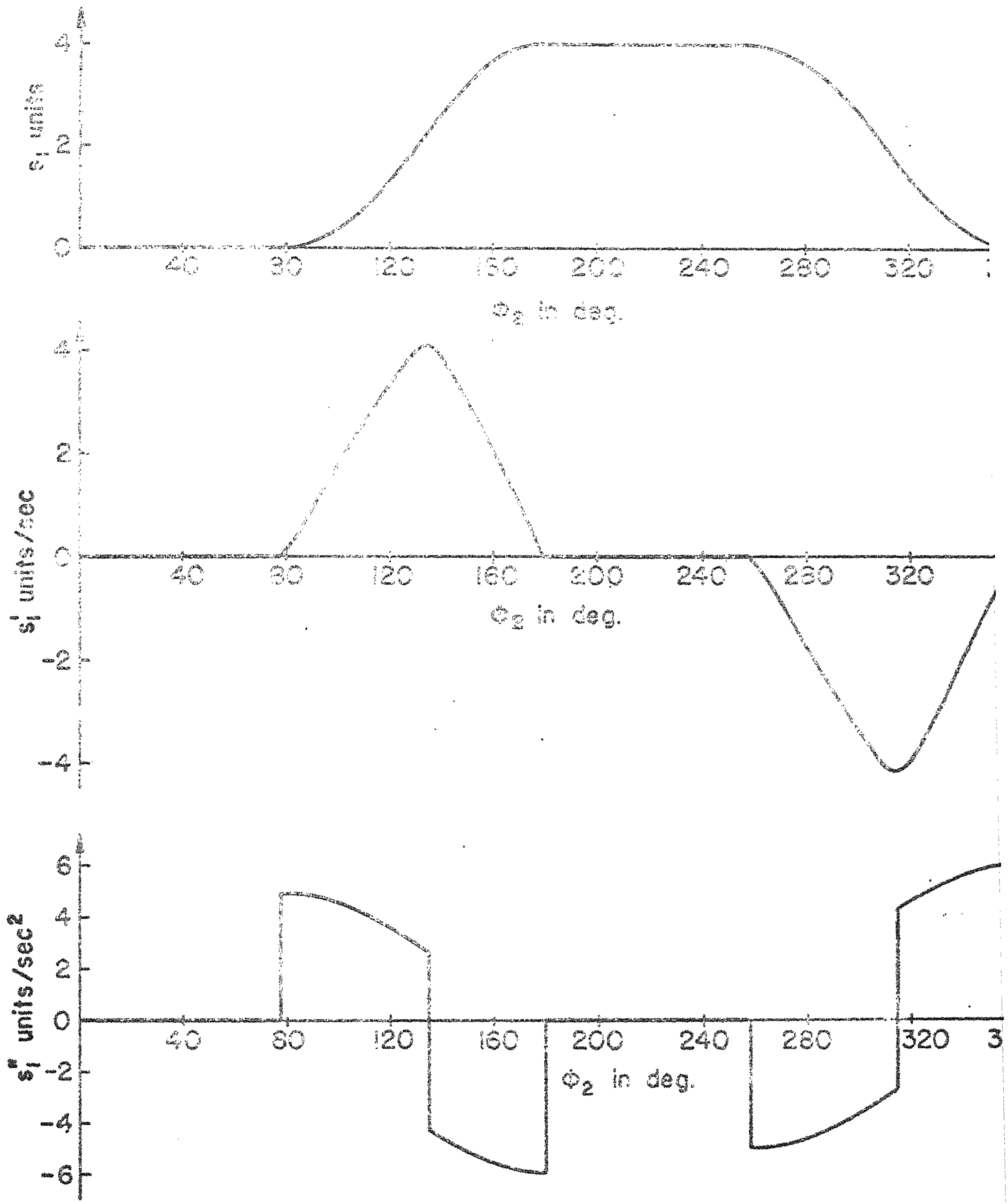


Fig. 2.4 Displacement, velocity and acceleration curves of type cam profile with an on-centre translating follower

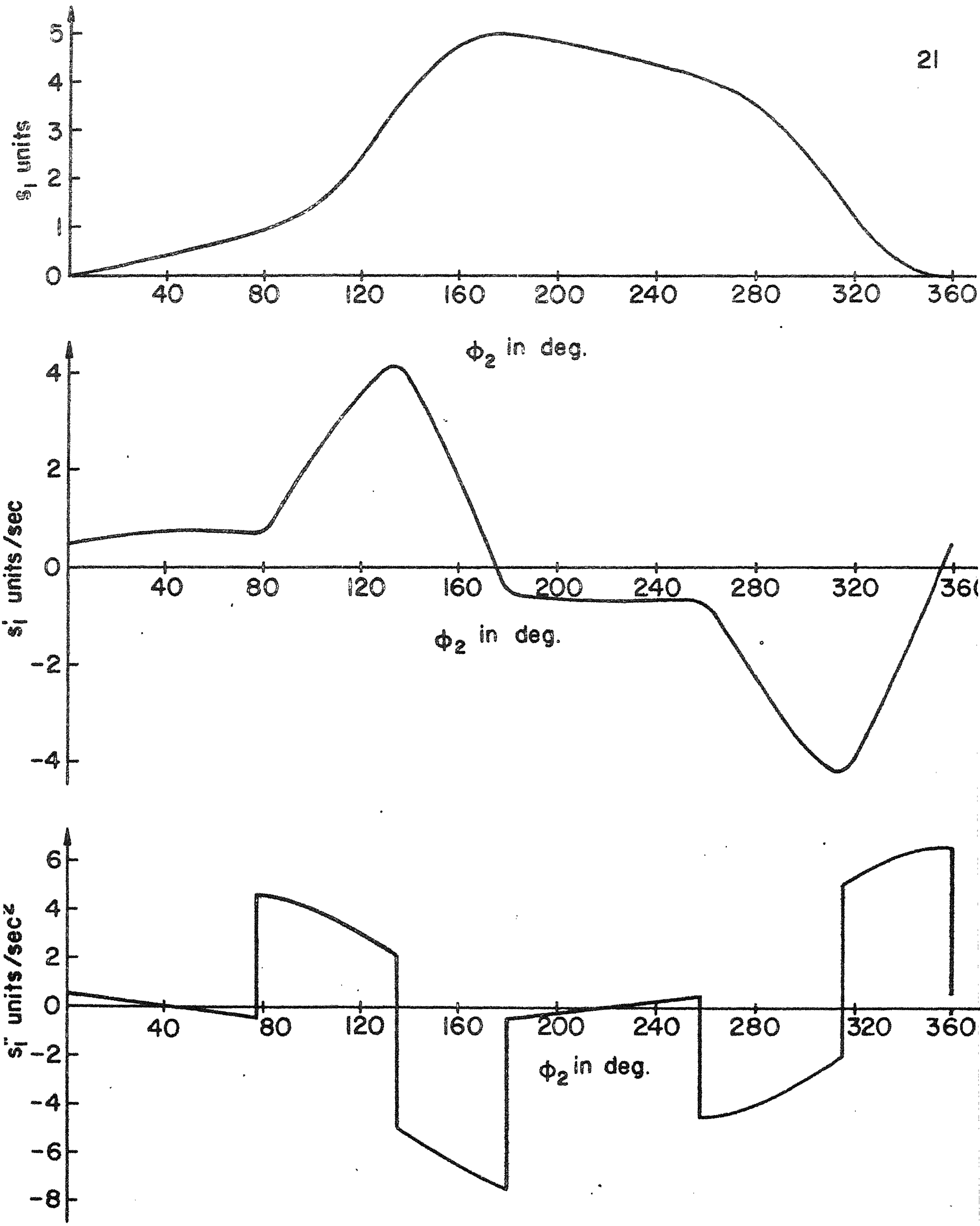


Fig.2.5 Displacement, velocity and acceleration curves of type I

Chapter 3

KINEMATIC ANALYSIS OF CONSTANT-BREADTH CAM-FOLLOWER MECHANISMS - TYPE II

3.1 General Remarks

The cam profile, chosen in Chapter 2, is comprising of circular arcs joining each other tangentially. The points where two such arcs are joined, the curvature of the profile changes abruptly. This leads to discontinuity in the acceleration of the output. The abrupt change of acceleration can be eliminated by replacing the triangle with a three cusped hypocycloid (deltoid)[1].

A three cusped hypocycloid can be generated as a locus of a point A lying on the circumference of a circle of radius R, when the circle rolls without slipping internally inside a fixed circle of diameter '1.5R'(Figure 3.1). When a rod BC of length 'ℓ' rolls over the hypocycloid the ends B and C generate the required constant-breadth profile. As mentioned by Hunt [1] - "The constant-breadth profile is an involute of the hypocycloid; also it may be considered as the envelope of one or both of the lines attached at B and C at right angles to the rolling rod".

3.2 Type II Cam Profile

Consider a hypocycloid of three cusps generated by a pair of circles having radii R and 1.5R respectively as shown in Figure 3.1. At point A the rod BC is tangential to the

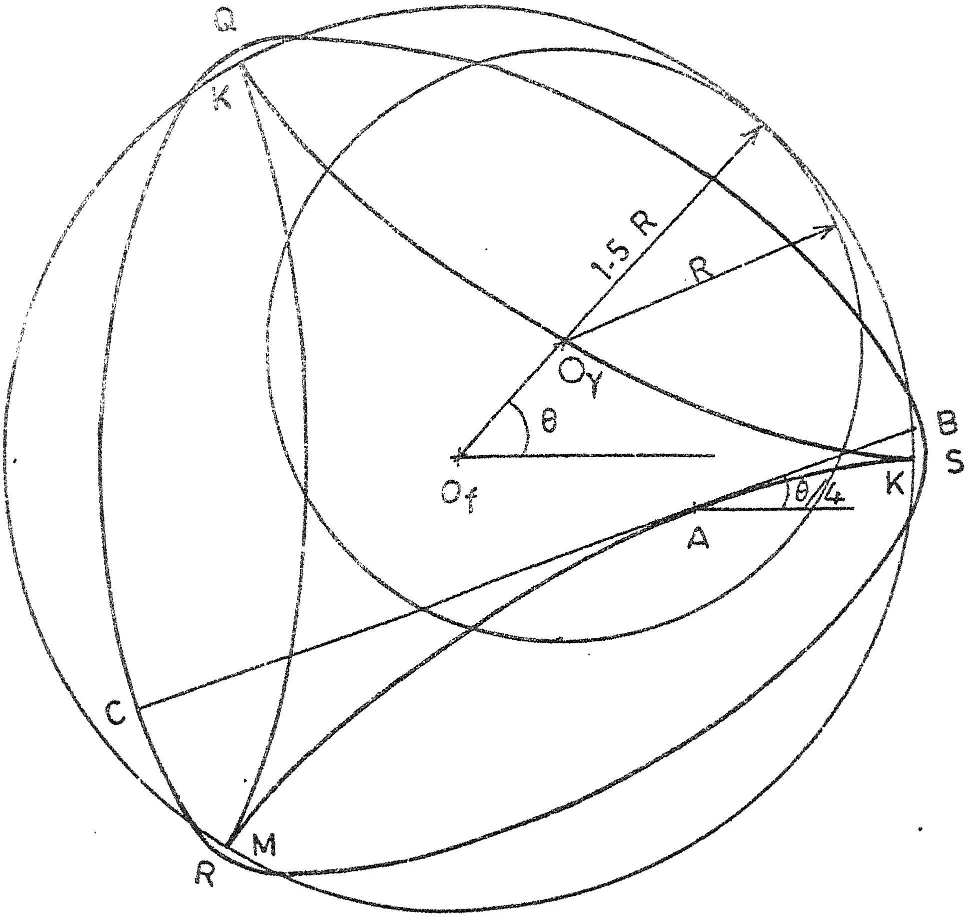


FIGURE 3.1

Type II Cam profile.

hypocycloid. If θ is the angle made by the line of centres, O_f, O_r, of the two circles with reference to horizontal, then the angle made by the rod BC with reference to horizontal is $\frac{\theta}{4}$. The parametric equations describing the coordinates of point $A(x_{2A}, y_{2A})$ in a coordinate system $X_2O_2Y_2$ will be

$$\begin{aligned} x_{2A} &= R\cos\theta + 2R\cos\frac{\theta}{2} \\ y_{2A} &= R\sin\theta - 2R\sin\frac{\theta}{2} \end{aligned} \qquad (3.1)$$

The arc length of each of the three curves in the hypocycloid is 5.333R. Let ℓ be the length of the rod such that

$$\ell = 2d + 5.333R \quad (d>0)$$

where d is the length of the extension of the rod beyond cusp point when the rod is tangential to the curve at the cusp point.

The equations describing the constant-breadth profile generated by points B and C are given by

$$\begin{aligned} x_{2B} &= R(\cos\theta+2\cos\frac{\theta}{2}) + (d+s)\cos\frac{\theta}{4} \\ y_{2B} &= R(\sin\theta-2\sin\frac{\theta}{2}) + (d+s)\sin\frac{\theta}{4} \end{aligned} \qquad (3.2)$$

$$\begin{aligned} x_{2C} &= R(\cos\theta+2\cos\frac{\theta}{2}) + (D-d-s)\cos(\pi+\frac{\theta}{4}) \\ y_{2C} &= R(\sin\theta-2\sin\frac{\theta}{2}) + (D-d-s)\sin(\pi+\frac{\theta}{4}) \end{aligned} \qquad (3.3)$$

where

$D = 2d + \frac{16}{3}R$ and $s = \frac{8}{3}R(1-\cos\frac{3\theta}{4})$ and θ varies from 0 to 4π.

Since points B and C generate the same profile, the profile generated by point B only is considered for further

discussion. The radius vector $\underset{\sim}{R}_2^{(P)}$ and the unit normal vector $\underset{\sim}{n}_2^{(P)}$ for a generic point P are given by,

$$\underset{\sim}{R}_2^{(P)} = [x_{2B}, y_{2B}, 1]^T,$$
$$\underset{\sim}{n}_2^{(P)} = [n_{2x}, n_{2y}]^T$$

where x_{2B} and y_{2B} are given by Equation (3.2), n_{2x} and n_{2y} are $\cos\frac{\theta}{4}$ and $\sin\frac{\theta}{4}$ respectively. Hence onwards, the profile described in this section will be referred to as Type II constant-breadth cam profile.

3.3 Kinematic Analysis of Translating, On-centre Follower

Consider a constant-breadth cam mechanism of Type II with a translating follower as shown in Figure 3.2a. The profile of the cam is described in Section 3.2. In this case it is assumed that the cam is rotating with a uniform angular velocity ω_2, with an angular rotation ϕ_2 as the parameter of the cam motion. The cam is rotating with the centre of fixed circle O_f as the pivot and at an initial position, line O_fKP is along the direction of translation of the follower. The linear displacement of the follower is measured from its initial position when the contact is maintained at P. Coordinate systems used in the present analysis are shown in Figure 3.2b. The required transformation matrices are given in Appendix II .

In this section, the expressions for displacement, velocity and acceleration of the follower are derived in parametric form, when the cam is rotating with a given uniform angular velocity, and assuming that the cam and the follower profiles are known.

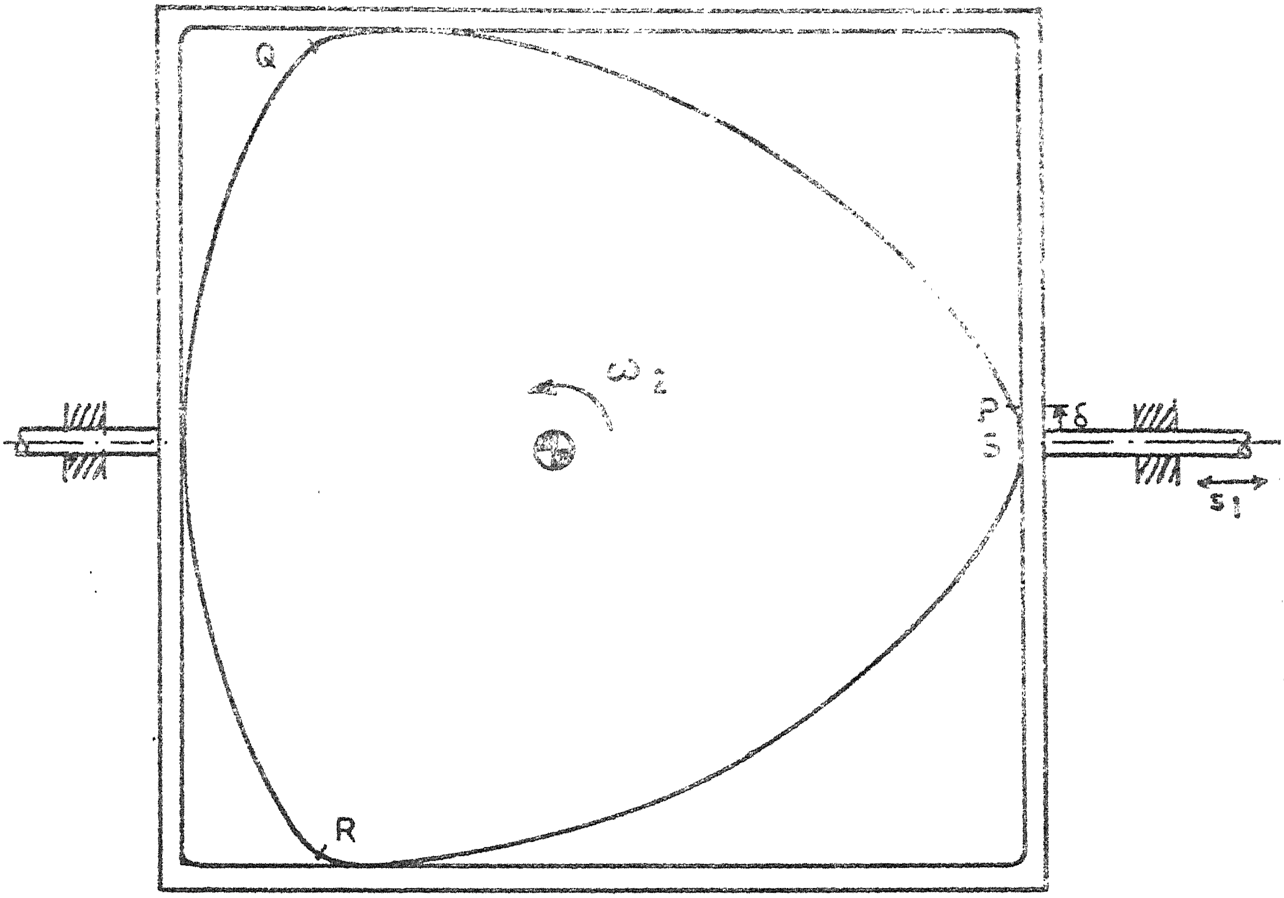


FIGURE 3.2 a

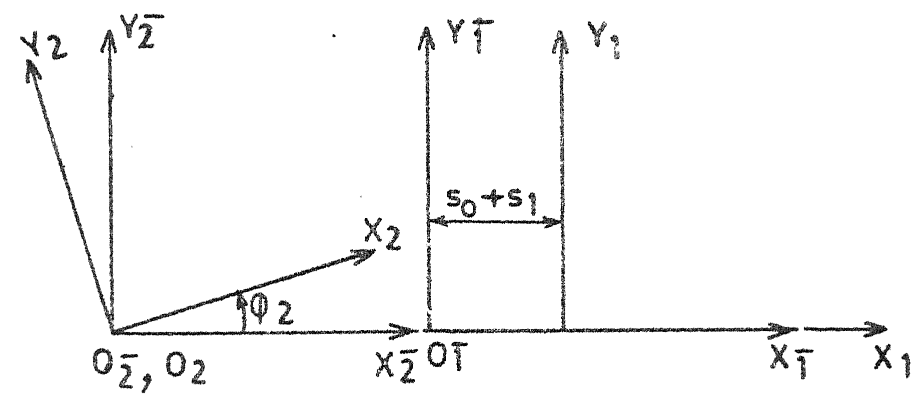


FIGURE 3.2 b

ype II Cam profile with on-centre translating follower

Following the procedure of kinematic analysis used in Section 2.2, with the same notations, Equation (2.1) gives a set of three independent equations for this case also. These are as follows:

$$x_{2B}\cos\phi_2 - y_{2B}\sin\phi_2 = s_o + s_1$$
$$x_{2B}\sin\phi_2 + y_{2B}\cos\phi_2 = \delta \qquad (3.4)$$
$$\frac{\theta}{4} + \phi_2 = 2\pi$$

where the initial displacement, $s_o = 3R+d$.

Solving the above set of Equations (3.4) for θ, δ and s_1, we have,

$$\theta = 2\pi - 4\phi_2 \qquad (3.5a)$$
$$\delta = x_{2B}\sin\phi_2 + y_{2B}\cos\phi_2 \qquad (3.5b)$$
$$s_1 = x_{2B}\cos\phi_2 - y_{2B}\sin\phi_2 - (3R+d) \qquad (3.5c)$$

where x_{2B}, y_{2B} are defined by Equation (3.2).

For a given value of ϕ_2, Equation (3.5a) gives the value of θ. Since θ and ϕ_2 are known, x_{2B}, y_{2B} can be computed. Then using Equation (3.5c), the displacement s_1 can be computed. The first and second derivatives of s_1 are termed as the velocity ratio and the acceleration ratio respectively. The expressions for the displacement, the velocity ratio and the acceleration ratio, in terms of the angular position, ϕ_2, are as follows:

$$s_1 = \frac{R}{3}(\cos 3\phi_2 - 1)$$
$$s_1' = -R\sin 3\phi_2 \qquad (3.6)$$

where

$$s_1' = \frac{ds_1}{d\phi_2} \qquad \text{and} \qquad s_1'' = \frac{d^2 s_1}{d\phi_2^2} .$$

The actual velocity and acceleration defined in Equation (2.4), Section 2.2 holds good for this case also.

3.4 Kinematic Analysis of an Oscillating Follower

Figure 3.3a shows the schematic diagram of a Type II constant-breadth cam with an oscillating follower and Figure 3.3b shows the coordinate systems used in the analysis. The cam profile is still the same as described in Section 3.2, and the parametric equations of the hypocycloid and the cam profile are as given in Equations (3.1) and (3.2) respectively. For the oscillating follower the pivot is chosen at a distance 'H' from the fixed circle centre O_f.

For the present case, at the point of contact P, following conditions will hold good.

$$\begin{aligned} [M_{\bar{2}2}]\underset{\sim}{R}_2^{(P)} &= [M_{\bar{2}1}]\underset{\sim}{R}_1^{(P)} \\ [L_{\bar{2}2}]\underset{\sim}{n}_2^{(P)} &= [L_{\bar{2}1}]\underset{\sim}{n}_1^{(P)} \end{aligned} \qquad (3.7)$$

where

$$\begin{aligned} \underset{\sim}{R}_2^{(P)} &= [x_{2B},\ y_{2B},\ 1]^T , \\ \underset{\sim}{n}_2^{(P)} &= [\cos\tfrac{\theta}{4},\ \sin\tfrac{\theta}{4}]^T , \\ \underset{\sim}{R}_1^{(P)} &= [x_1,\ y_1,\ 1]^T , \\ \underset{\sim}{n}_1^{(P)} &= [1,\ 0]^T . \end{aligned}$$

Here x_{2B} and y_{2B} are defined in Equation (3.2),

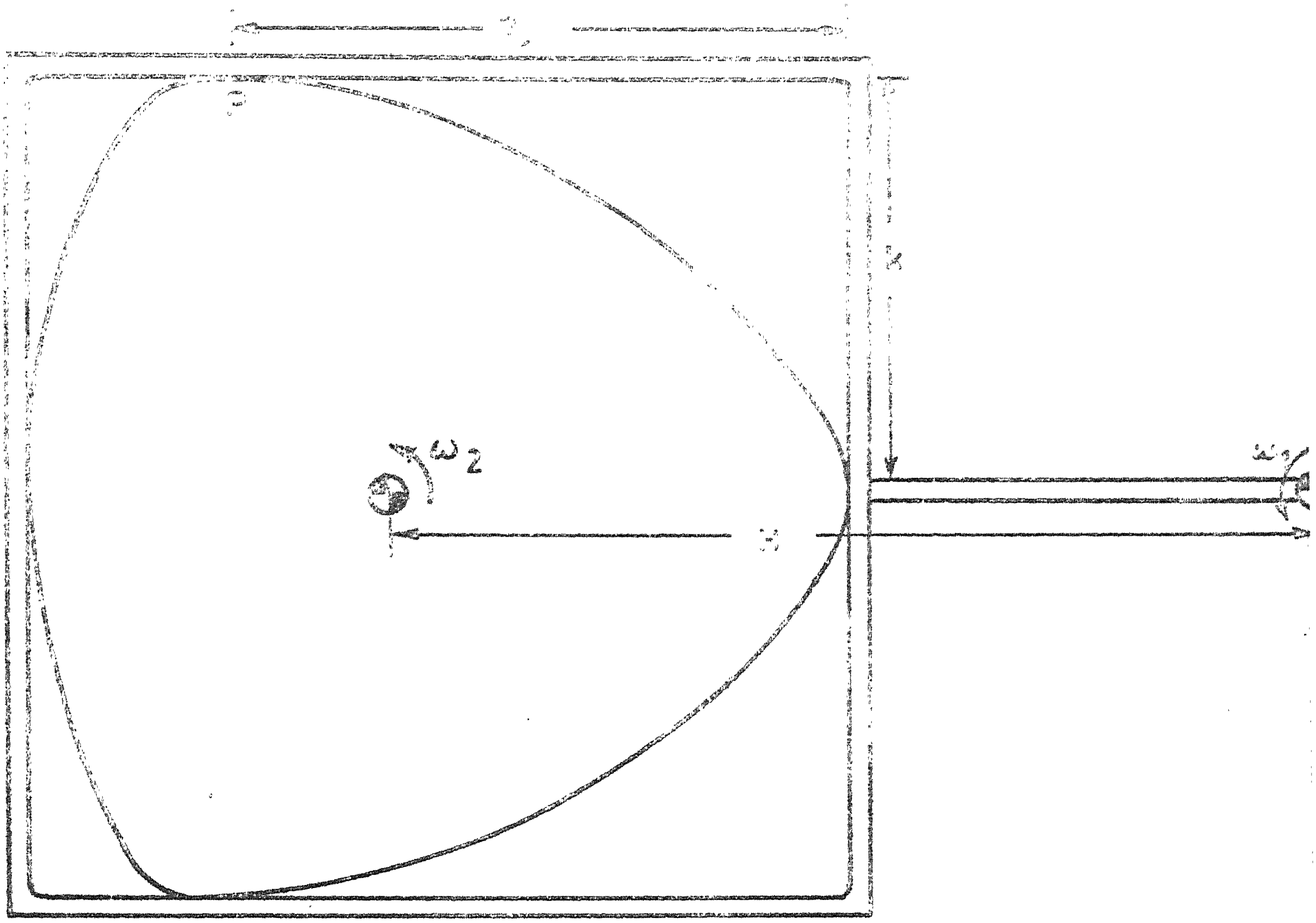


FIGURE 3.3 a

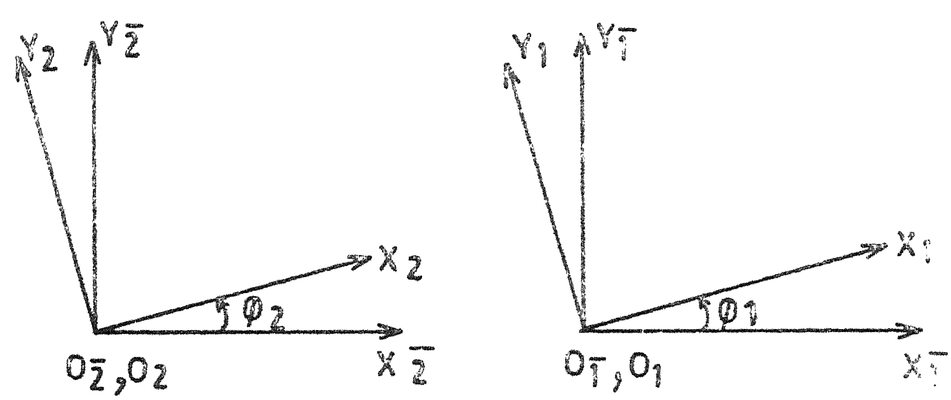


FIGURE 3.3 b

$x_1 = -(H-1.5R-d) - \eta$, $y_1 = k$ where k is given, and η is a parameter indicating the location of the point of contact on the follower profile and the transformation matrices are given in Appendix II.

After substituting appropriate expressions of x_{2B}, y_{2B}, x_1, y_1 etc., we get,

$$R\cos(\theta+\phi_2) + 2R\cos(\phi_2-\tfrac{\theta}{2}) + (d+s)\cos(\tfrac{\theta}{4}+\phi_2)$$
$$= -(h+\eta)\cos\phi_1 - k\sin\phi_1 + H \tag{3.8a}$$

$$R\sin(\theta+\phi_2) + 2R\sin(\phi_2-\tfrac{\theta}{2}) + (d+s)\sin(\tfrac{\theta}{4}+\phi_2)$$
$$= -(h+\eta)\sin\phi_1 + k\cos\phi_1 \tag{3.8b}$$

$$\theta = 4(\phi_1 + \tfrac{\pi}{2} - \phi_2) \tag{3.8c}$$

where $h = H-1.5R-d$. Eliminating θ and η from the above set, we get,

$$\frac{R}{3}\sin(3\phi_2-3\phi_1) - H\sin\phi_1 = d - k + \frac{8R}{3} \tag{3.9}$$

For a given value of ϕ_2, Equation (3.9) can be solved for the angular displacement of the follower, ϕ_1, assuming that the values of the parameters R, d, k, H are known. Starting with a initial trial value of ϕ_1, say ϕ_{1i}, an improved value of ϕ_1 can be computed using the Newton-Raphson formula,

$$\phi_{1(i+1)} = \phi_{1i} - \frac{f(\phi_{1i})}{f'(\phi_{1i})}$$

That the value of ϕ_1 is considered to be satisfactory if the difference between two successive values in the iteration process is less than 10^{-5} radians. Once the value of ϕ_1 is

obtained the velocity ratio and the acceleration ratio can be computed using the following relations.

$$\phi_1' = \frac{\cos(3\phi_2 - 3\phi_1)}{\cos(3\phi_2 - 3\phi_1) + \frac{H}{R}\cos\phi_1} \tag{3.10}$$

$$\phi_1'' = \frac{\frac{3H}{R}(\phi_1' - 1)\cos\phi_1 \sin(3\phi_2 - 3\phi_1) + \frac{H}{R}\phi_1' \sin\phi_1 \cos(3\phi_2 - 3\phi_1)}{\left\{\cos(3\phi_2 - 3\phi_1) + \frac{H}{R}\cos\phi_1\right\}^2} \tag{3.11}$$

where

$$\phi_1' = \frac{d\phi_1}{d\phi_2} \quad \text{and} \quad \phi_1'' = \frac{d^2\phi_1}{d\phi_2^2} .$$

This concludes the kinematic analysis procedure for the Type II oscillating cam-follower mechanism.

3.5 Numerical Examples

Example # 1

To illustrate the procedure of kinematic analysis presented in Section 3.3, for the on-centre translating follower, a cam profile with following data was selected. R = 1 unit, d = 0.1R unit.

The hypocycloid and the profile generated are shown in Figure 3.1. A computer programme was developed to compute displacement, velocity and acceleration of the follower. The uniform angular velocity is assumed to be unity. The step size for angle ϕ_2 was taken to be one degree. The curves showing variations of the displacement, velocity and acceleration of the follower as a function of ϕ_2, the angle of rotation, are shown in Figure 3.4.

Example # 2

To illustrate the procedure of kinematic analysis presented in Section 3.4, for the oscillating follower, a cam profile with following data was selected. R = 1 unit, d = 0.1R unit, k = 2.766 units and H = 6R units.

The hypocycloid and the profile generated are shown Figure 3.1. A computer programme was developed to compute angular rotation, angular velocity and angular acceleration of the follower, assuming ω_2 is unity. The step size of ϕ_2 was also unity. The curves showing variations of angular rotation, angular velocity and angular acceleration of the follower as a function of ϕ_2, the angle of motion, are shown in Figure 3.5.

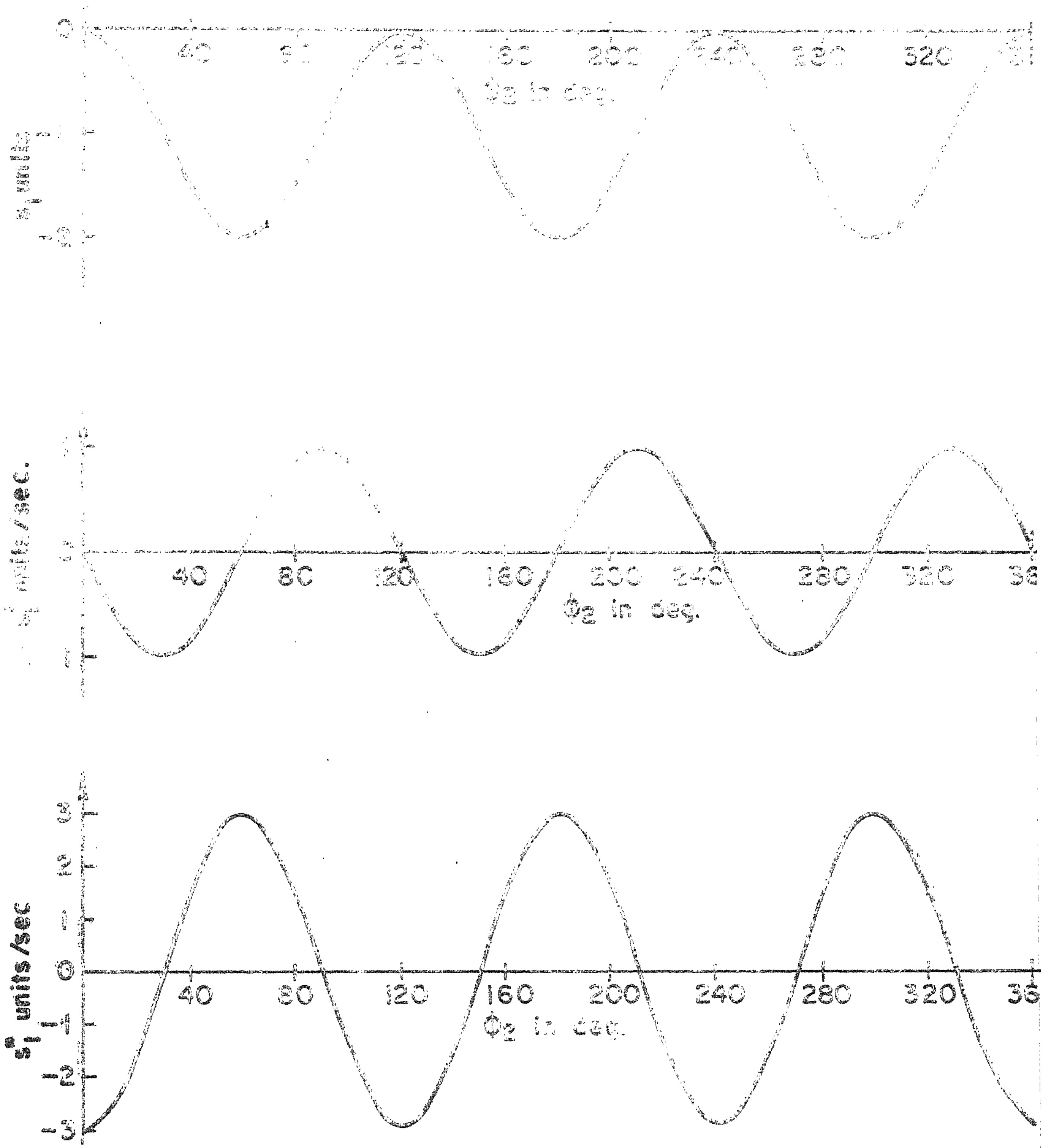


Fig. 3.4 Linear displacement, velocity and acceleration curves of Type II cam profile with translating follower.

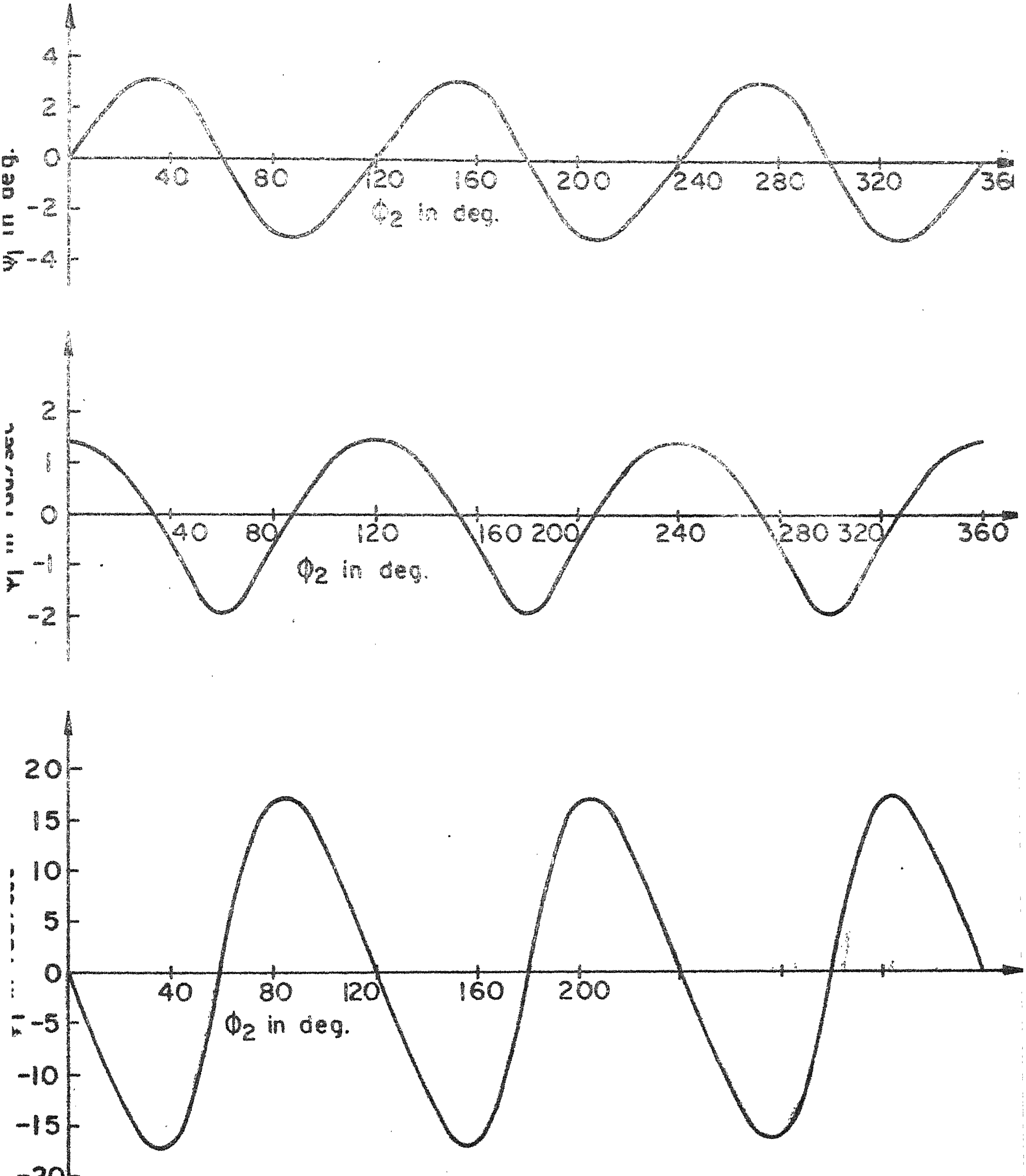


Fig. 3.5 Angular displacement, angular velocity and angular acceleration curves of type II cam profile with an oscillating follower.

Chapter 4

KINEMATIC ANALYSIS OF TYPE I AND TYPE II CAM-FOLLOWER MECHANISMS - GENERAL CASES

4.1 Type I Cam Mechanism, General Case

The cam profile chosen in this section is the same as described in Section 2.1. Figure 4.1a shows a schematic diagram of a translating as well as an oscillating follower being driven by a Type I cam profile. The cam is assumed to be rotating with a uniform angular velocity ω_2; the parameter of motion being the angular rotation of cam ϕ_2. The follower motion can be described by means of two parameters, namely, the angular rotation ϕ_1 of the follower link DL and the linear displacement s_1 along the direction of the slot of the follower link DL. The simultaneous rotation and translation of the follower link is constrained by a slotted pivot which is at a distance h from the centre of rotation of the cam.

The follower profile consists of two flat faces which are mutually perpendicular to each other and one of these surfaces is parallel to the follower arm DL. The cam surface contacts the follower surfaces at two points namely P and Q. The kinematic analysis procedure is aimed at deriving the expressions for s_1 and ϕ_1 using following conditions at the contacting points.

At point P

$$\begin{aligned} [M_{\bar{2}2}]\underset{\sim}{R}_2^{(P)} &= [M_{\bar{2}1}]\underset{\sim}{R}_1^{(P)} \\ [L_{\bar{2}2}]\underset{\sim}{n}_2^{(P)} &= [L_{\bar{2}1}]\underset{\sim}{n}_1^{(P)} \end{aligned} \qquad (4.1)$$

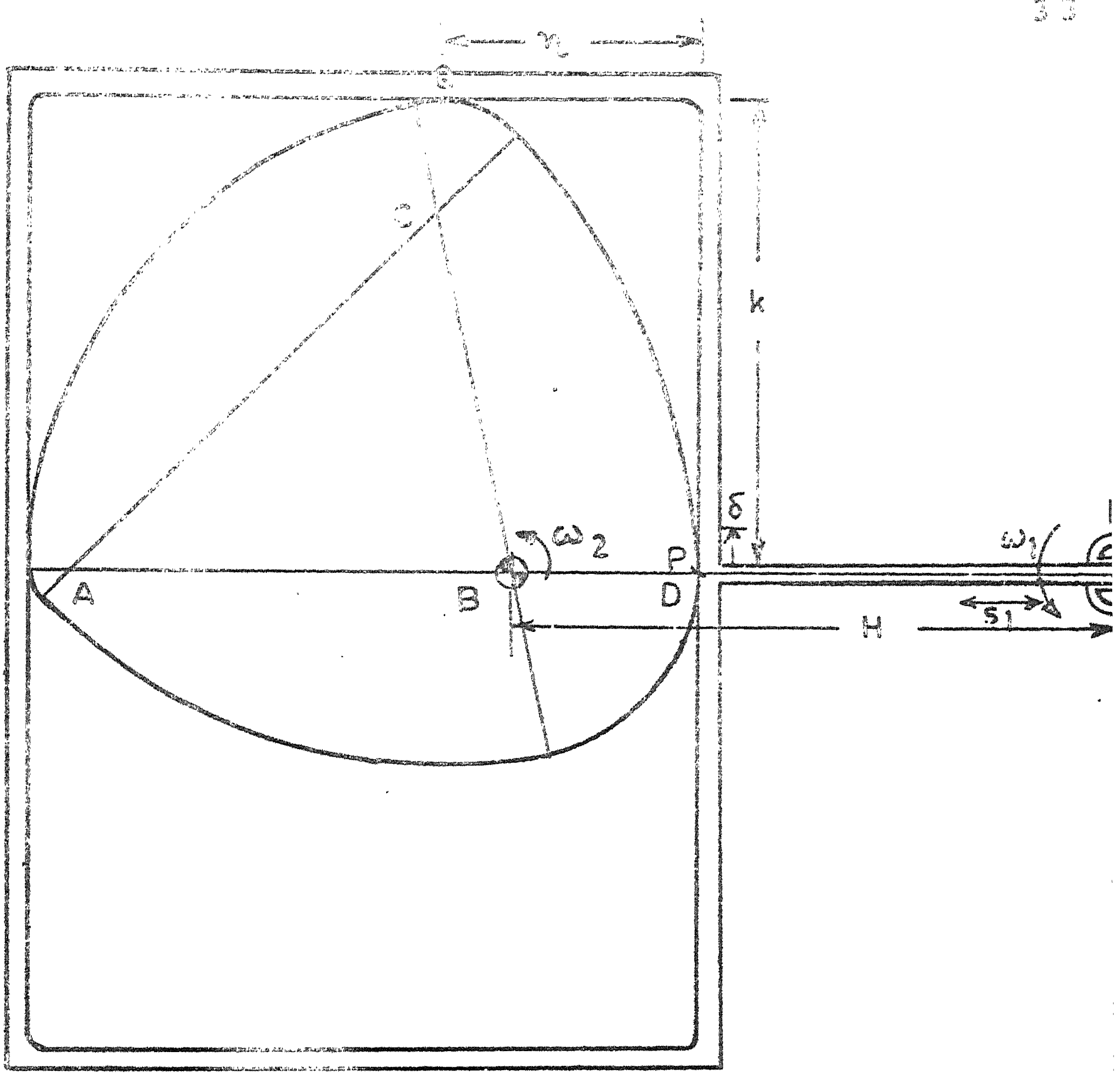


FIGURE 4.1 a

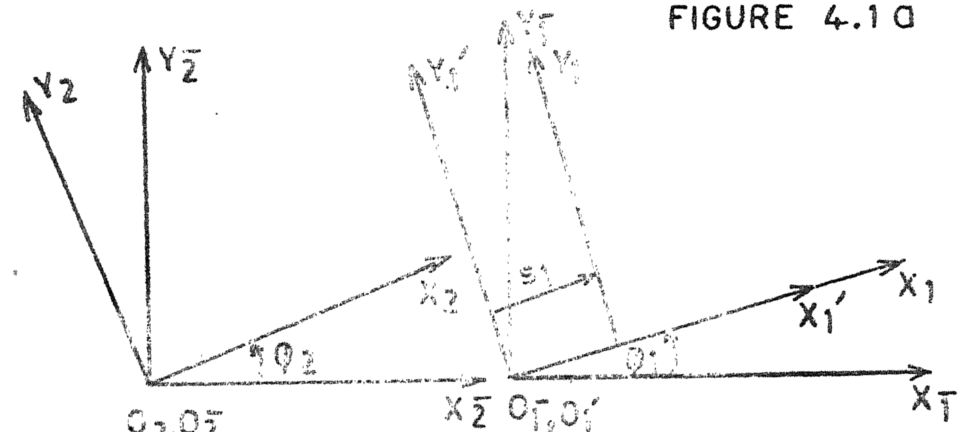


FIGURE 4.1 b

Type I Cam profile General case.

where

$$\underset{\sim}{R}_1^{(P)} = [d-h, \delta , 1]^T$$
$$\underset{\sim}{R}_2^{(P)} = [x_{2P}, y_{2P}, 1]^T$$
$$\underset{\sim}{n}_1^{(P)} = [1, 0]^T$$
$$\underset{\sim}{n}_2^{(P)} = [\cos(\pi+\theta_P), \sin(\pi+\theta_P)]^T$$
$$x_{2P} = k_1 + k_2\cos\theta_P$$
$$y_{2P} = k_3 + k_2\sin\theta_P$$

and δ is linear parameter shown in Figure 4.1a.

At point Q

$$[M_{\bar{2}2}]\underset{\sim}{R}_2^{(Q)} = [M_{\bar{2}1}]\underset{\sim}{R}_1^{(Q)}$$
$$[L_{\bar{2}2}]\underset{\sim}{n}_2^{(Q)} = [L_{\bar{2}1}]\underset{\sim}{n}_1^{(Q)} \qquad (4.2)$$

where

$$\underset{\sim}{R}_1^{(Q)} = [d-h-\eta, k, 1]^T$$
$$\underset{\sim}{R}_2^{(Q)} = [x_{2Q}, y_{2Q}, 1]^T$$
$$\underset{\sim}{n}_1^{(Q)} = [0, 1]^T$$
$$\underset{\sim}{n}_2^{(Q)} = [\cos(\pi+\theta_Q), \sin(\pi+\theta_Q)]^T$$
$$x_{2Q} = k_4 + k_5\cos\theta_Q$$
$$y_{2Q} = k_6 + k_5\sin\theta_Q$$

and η is linear parameter shown in Figure 4.1a.

Detailed expressions for the transformation matrices mentioned in Equations (4.1) and (4.2) are given in Appendix III. Since the Type I cam profile consists of six different sections, it is not known 'apriori' as to which sections are contacting

at the points of contact P and Q. However, all the sections can be expressed in a common form as mentioned above. Expressions for k_1, k_2, k_3, k_4, k_5 and k_6 for all the sections are given in Table 4.1.

To begin with, it can be ascertained from the initial conditions, which profiles are contacting the points P and Q. Then as the cam rotates, these profiles move and the values of the angular parameter θ_P and θ_Q defining the points of P and Q vary such that after a certain amount of angular rotation contact is no longer maintained with the same profile, but with the adjacent profile. This has been taken care of by checking the values of θ_P and θ_Q with respect to the limiting values of θ for all the sections as given in Table 4.1.

Equations (4.1) and (4.2) can also be written in the form,

$$\begin{aligned} k_1\cos\phi_2 + k_2\cos(\phi_2+\theta_P) - k_3\sin\phi_2 &= (d-h+s_1)\cos\phi_1 + h - \delta\sin\phi_1 \\ k_1\sin\phi_2 + k_2\sin(\phi_2+\theta_P) + k_3\cos\phi_2 &= (d-h+s_1)\sin\phi_1 + \delta\cos\phi_1 \end{aligned} \tag{4.3}$$

$$\begin{aligned} k_4\cos\phi_2 + k_5\cos(\phi_2+\theta_Q) - k_6\sin\phi_2 &= (d-h-\eta+s_1)\cos\phi_1 - k\sin\phi_1 + h \\ k_4\sin\phi_2 + k_5\sin(\phi_2+\theta_Q) + k_6\cos\phi_2 &= (d-h-\eta+s_1)\sin\phi_1 + k\cos\phi_1 \end{aligned} \tag{4.4}$$

$$\begin{aligned} \phi_1 &= \phi_2 + \theta_P + \pi \\ \phi_1 &= \phi_2 + \theta_Q - \frac{\pi}{2} \end{aligned} \tag{4.5}$$

Section	$k_1 = k_4$	$k_2 = k_5$	$k_3 = k_6$	θ_P and θ_Q Range
I	0	d	0	$2\pi-\beta \leq \theta \leq 2\pi$
II	$-b \cos\beta$	b+d	$b \sin\beta$	$\pi+\alpha \leq \theta \leq 2\pi-\beta$
III	$-a$	d+b−c	0	$\pi \leq \theta \leq \pi+\alpha$
IV	0	a+d+b−c	0	$\pi-\beta \leq \theta \leq \pi$
V	$-b \cos\beta$	a+d−c	$b \sin\beta$	$\alpha \leq \theta \leq \pi-\beta$
VI	$-a$	a+d	0	$0 \leq \theta \leq \alpha$

Table 4.1 Values of k_1, k_2, k_3, k_4, k_5 and k_6 of Type I cam profile for General case.

Equations (4.5), after eliminating ϕ_1, give

$$\theta_P = \theta_Q - \frac{3\pi}{2} \tag{4.6}$$

Equations (4.4), after eliminating η and s_1, give

$$\theta_Q = 2\tan^{-1}\left\{\frac{-(k_6+h\sin\phi_2) \pm \{k_6^2+k_4^2+h^2+2h(k_6\sin\phi_2-k_4\cos\phi_2)-(k_5-k)^2\}^{\frac{1}{2}}}{(k_5-k-k_4+h\cos\phi_2)}\right\} \tag{4.7}$$

Equations (4.3) give,

$$s_1 = k_2 + h - d + (k_1 - h\cos\phi_2)\cos\theta_P + (k_3 + h\sin\phi_2)\sin\theta_P \tag{4.8}$$

Differentiating Equations (4.5) and (4.8) with respect to ϕ_2, the velocity ratio expressions are as follows:

$$\phi_1' = 1 + \theta_Q' \tag{4.9}$$

$$s_1' = \{(k_3\cos\theta_P - k_1\sin\theta_P) + h\sin(\phi_2+\theta_P)\}\theta_P' + h\sin(\phi_2+\theta_P) \tag{4.10}$$

where θ_Q' and θ_P' are given in Appendix III. Differentiating Equations (4.9) and (4.10) with respect to ϕ_2, once again, the acceleration ratio expressions are as follows:

$$\phi_1'' = \theta_Q'' \tag{4.11}$$

$$s_1'' = h(1+2\theta_P'+\theta_P'^2+\theta_P'')\cos(\phi_2+\theta_P) - \theta_P'^2(k_1\cos\theta_P+k_3\sin\theta_P)$$
$$+ \theta_P''(k_3\cos\theta_P - k_1\sin\theta_P) \tag{4.12}$$

where θ_Q'' and θ_P'' are given in Appendix III.

This concludes the kinematic analysis procedure for the present case.

4.2 Type II Cam Mechanism, General Case

The cam profile chosen in this section is the same as described in Section 3.2. Figure 4.2a shows a schematic

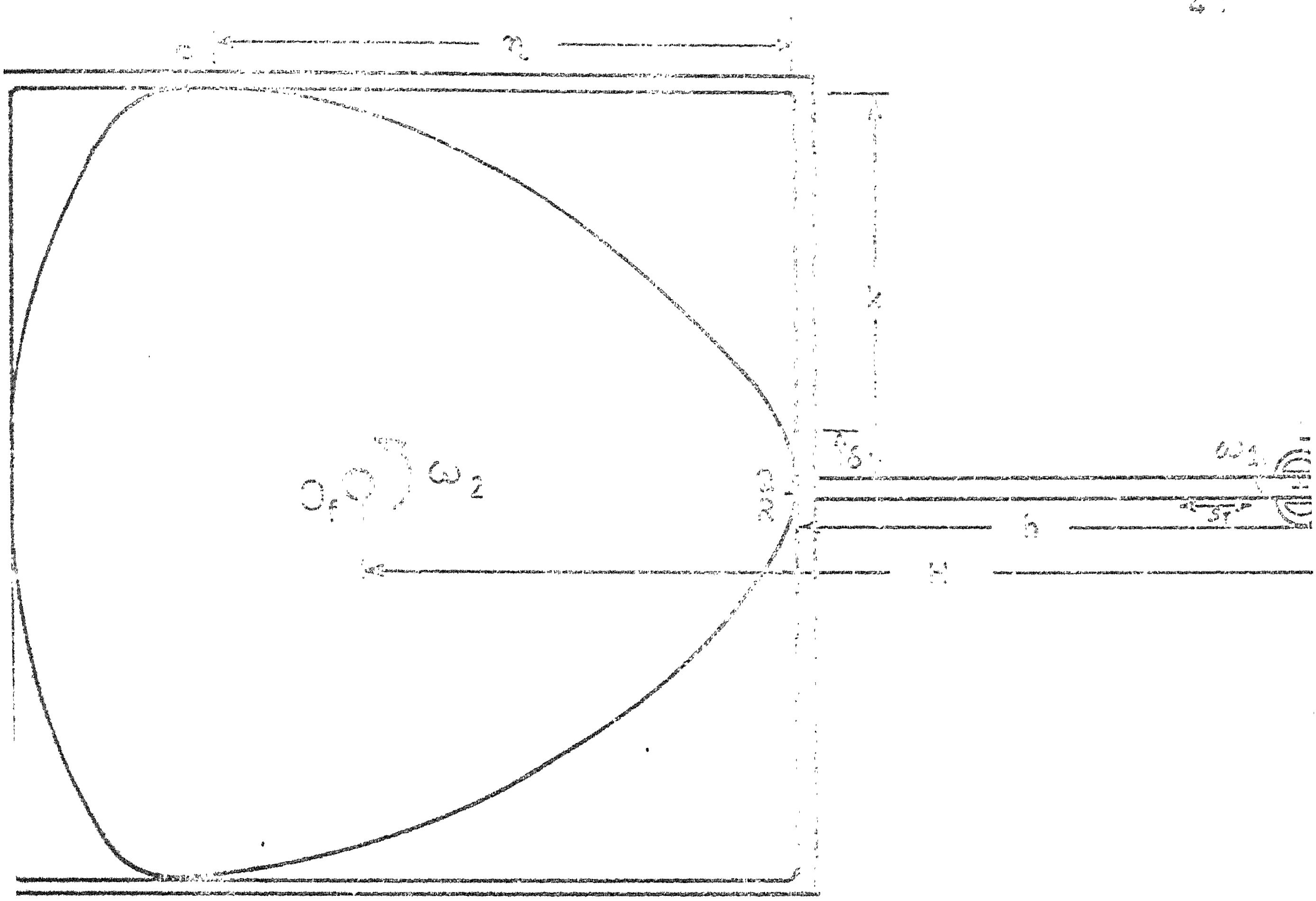


FIGURE 4.2a

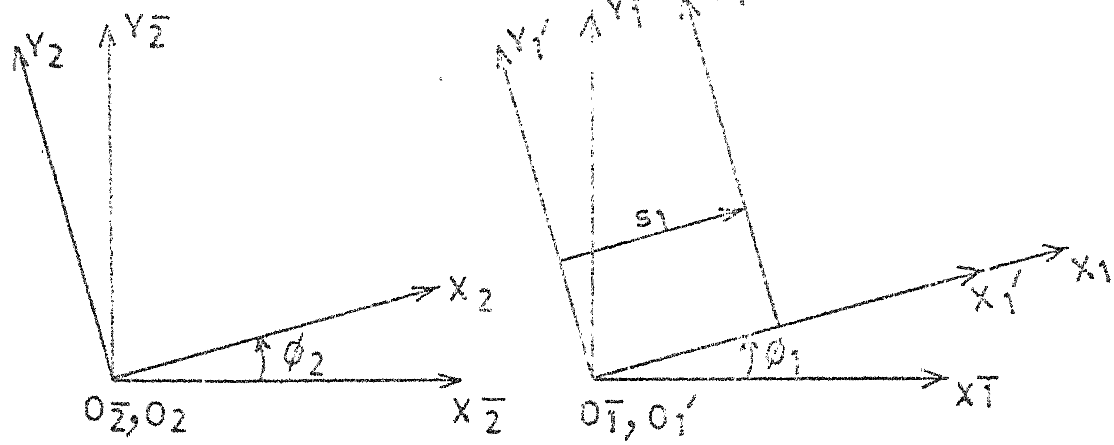


FIGURE 4.2b

Type II Cam profile

diagram of a translating as well as oscillating follower being driven by a Type II cam profile. The cam is assumed to be rotating with a uniform angular velocity ω_2; the parameter of motion being the angular rotation of cam ϕ_2. The follower motion is described by two parameters, namely, the angular rotation, ϕ_1, of the follower link RL and the linear displacement s_1 of the follower link RL along the direction of the slot. The simultaneous rotation and translation of the follower link is constrained by a slotted pivot which is at a distance H from the centre of rotation of the cam, O_f.

The follower profile consists of two flat faces which are mutually perpendicular to each other and one of these surfaces is parallel to the follower arm RL. The cam surface contacts the follower surfaces at two different points P and Q.

Following the same procedure of kinematic analysis used in Section 4.1, at point P Equations (4.1) can be rewritten as follows:

$$x_{2P}\cos\phi_2 - y_{2P}\sin\phi_2 = -(h-\eta)\cos\phi_1 - k\sin\phi_1 + (s_o+s_1)\cos\phi_1 + H \tag{4.13a}$$

$$x_{2P}\sin\phi_2 + y_{2P}\cos\phi_2 = -(h-\eta)\sin\phi_1 + k\cos\phi_1 + (s_o+s_1)\sin\phi_1 \tag{4.13b}$$

$$\phi_1 = \phi_2 + \frac{\theta_P}{4} - \frac{\pi}{2} \tag{4.14}$$

where, in this case,

$$\underset{\sim}{R}_1^{(P)} = [-(h-\eta),\ k,\ 1]^T$$

$$\underset{\sim}{R}_2^{(P)} = [x_{2P},\ y_{2P},\ 1]^T$$

$$\underset{\sim}{n}_1^{(P)} = [0,\ 1]^T$$

$$\underset{\sim}{n}_2^{(P)} = [\cos\frac{\theta_P}{4},\ \sin\frac{\theta_P}{4}]^T$$

$$x_{2P} = R(\cos\theta_P+2\cos\frac{\theta_P}{2}) + (d+s)\cos\frac{\theta_P}{4}$$

$$y_{2P} = R(\sin\theta_P-2\sin\frac{\theta_P}{2}) + (d+s)\sin\frac{\theta_P}{4}$$

and η is linear parameter shown in Figure 4.2a.

At point Q, Equations (4.2) can be rewritten as follows:

$$x_{2Q}\cos\phi_2-y_{2Q}\sin\phi_2 = -h\cos\phi_1-\delta\sin\phi_1+(s_o+s_1)\cos\phi_1+H \quad (4.15a)$$

$$x_{2Q}\sin\phi_2+y_{2Q}\cos\phi_2 = -h\sin\phi_1+\delta\cos\phi_1+(s_o+s_1)\sin\phi_1 \quad (4.15b)$$

$$\phi_1 = \phi_2 + \frac{\theta_Q}{4} \quad (4.16)$$

where

$$\underset{\sim}{R}_1^{(Q)} = [-h, \delta , 1]^T$$

$$\underset{\sim}{R}_2^{(Q)} = [x_{2Q}, y_{2Q}, 1]^T$$

$$\underset{\sim}{n}_1^{(Q)} = [1, 0]^T$$

$$\underset{\sim}{n}_2^{(Q)} = [\cos\frac{\theta_Q}{4}, \sin\frac{\theta_Q}{4}]^T$$

$$x_{2Q}= R(\cos\theta_Q+2\cos\frac{\theta_Q}{2}) + (d+s)\cos\frac{\theta_Q}{4}$$

$$y_{2Q}= R(\sin\theta -2\sin\frac{\theta_Q}{2}) + (d+s)\sin\frac{\theta_Q}{4}$$

and δ is linear parameter shown in Figure 4.2a and the required transformation matrices are given in Appendix III. Eliminating ϕ_1 from Equations (4.14) and (4.16) yields,

$$\theta_P= \theta_Q + 2\pi \quad (4.17)$$

Eliminating η from Equations (4.13a) and (4.13b), substituting for s also give,

$$\cos\frac{3\theta_P}{4} -\frac{3H}{R}\cos(\phi_2+\frac{\theta_P}{4}) + \frac{3}{R}(d+\frac{8R}{3}-k) = 0 \quad (4.18)$$

Eliminating δ from Equations (4.15a) and (4.15b), give

$$s_1 = \frac{R}{3}\cos\frac{3\theta_Q}{4} - H\cos(\phi_2+\frac{\theta_Q}{4}) + h + \frac{8R}{3} + d \quad (4.19)$$

For a given value of ϕ_2, Equation (4.18) can be solved for the angular position of the follower, θ_P, assuming the values of the parameters R, d, k, H are known, using the Newton-Raphson formula explained in Section 3.4. Once the value of θ_P is calculated, using Equation (4.14), ϕ_1 can be calculated.

The first and second derivates of Equations (4.14) and (4.19) give the velocity ratios and the acceleration ratios for angular and linear motions. The expressions are as follows:

$$\phi_1' = 1 + \{K_1/(K_2-K_3)\} \tag{4.20}$$

$$\phi_1'' = \{K_1'(K_2-K_3)-K_1(K_2'-K_3')\}/(K_2-K_3)^2 \tag{4.21}$$

where

$$\phi_1' = \frac{d\phi_1}{d\phi_2}, \quad \phi_1'' = \frac{d^2\phi_1}{d\phi_2^2} \text{ and } K_1, K_2, K_3, K_1', K_2' \text{ and } K_3'$$

are as given in Appendix III.

$$s_1' = \{-\frac{R}{4}\sin\frac{3\theta_Q}{4} + \frac{H}{4}\sin(\phi_2+\frac{\theta_Q}{4})\}\theta_Q' + H\sin(\phi_2+\frac{\theta_Q}{4}) \tag{4.22}$$

$$\begin{aligned} s_1'' = {} & \{-\frac{R}{4}\sin\frac{3\theta_Q}{4} + \frac{H}{4}\sin(\phi_2+\frac{\theta_Q}{4})\}\theta_Q'' \\ & + \{\theta_Q'(\frac{H}{16}\cos(\phi_2+\frac{\theta_Q}{4}) - \frac{3R}{4}\cos\frac{3\theta_Q}{4}) + \frac{H}{2}\cos(\phi_2+\frac{\theta_Q}{4})\}\theta_Q' \\ & + H\cos(\phi_2+\frac{\theta_Q}{4}) \end{aligned} \tag{4.23}$$

where

$$s_1' = \frac{ds_1}{d\phi_2} \text{ and } s_1'' = \frac{d^2 s_1}{d\phi_2^2}.$$

This concludes the kinematic analysis procedure for the present case.

4.3 Numerical Examples

Example # 1

To illustrate the procedure of kinematic analysis presented in Section 4.1, for the translating as well as oscillating follower, a Type I cam profile with following data was selected. a = 6 units, b = 5 units, c = 7 units, d = 2.5 units and h = 18 units.

A computer program was developed to compute displacement, velocity ratio, acceleration ratio, ϕ_1, ϕ_1' and ϕ_1'' of the follower. The distance between the pivot and B (one of the vertices of $\triangle ABC$) is 18 units. Provision is made in the program to select proper values of k_1, k_2, k_3, k_4, k_5 and k_6. The initial values of θ_P and θ_Q are taken as zero and 90 degrees respectively. The uniform angular velocity is assumed to be unity. The step size for angle ϕ_2 was taken to be one degree. The curves showing variations of displacement, velocity, acceleration, ϕ_1, ϕ_1' and ϕ_1'' of the follower as a function of ϕ_2, are shown in Figures 4.3a and 4.3b.

Example # 2

To illustrate the procedure of kinematic analysis presented in Section 4.2, for the translating as well as oscillating follower, a Type II cam profile with the following data was selected. R = 1 unit, d = 0.1R unit and H = 6R units.

A computer programme was developed to compute displacement, velocity, acceleration, ϕ_1, ϕ_1' and ϕ_1'' of the follower. The uniform angular velocity is assumed to be zero. To start with

the values of θ_P and θ_Q are assumed to be 360 degrees and zero degree respectively. The step size for angle ϕ_2 was taken to be one degree. The curves showing variations of displacement, velocity, acceleration, ϕ_1, ϕ_1' and ϕ_1'' of the follower as a function of ϕ_2 are shown in Figures 4.4a and 4.4b.

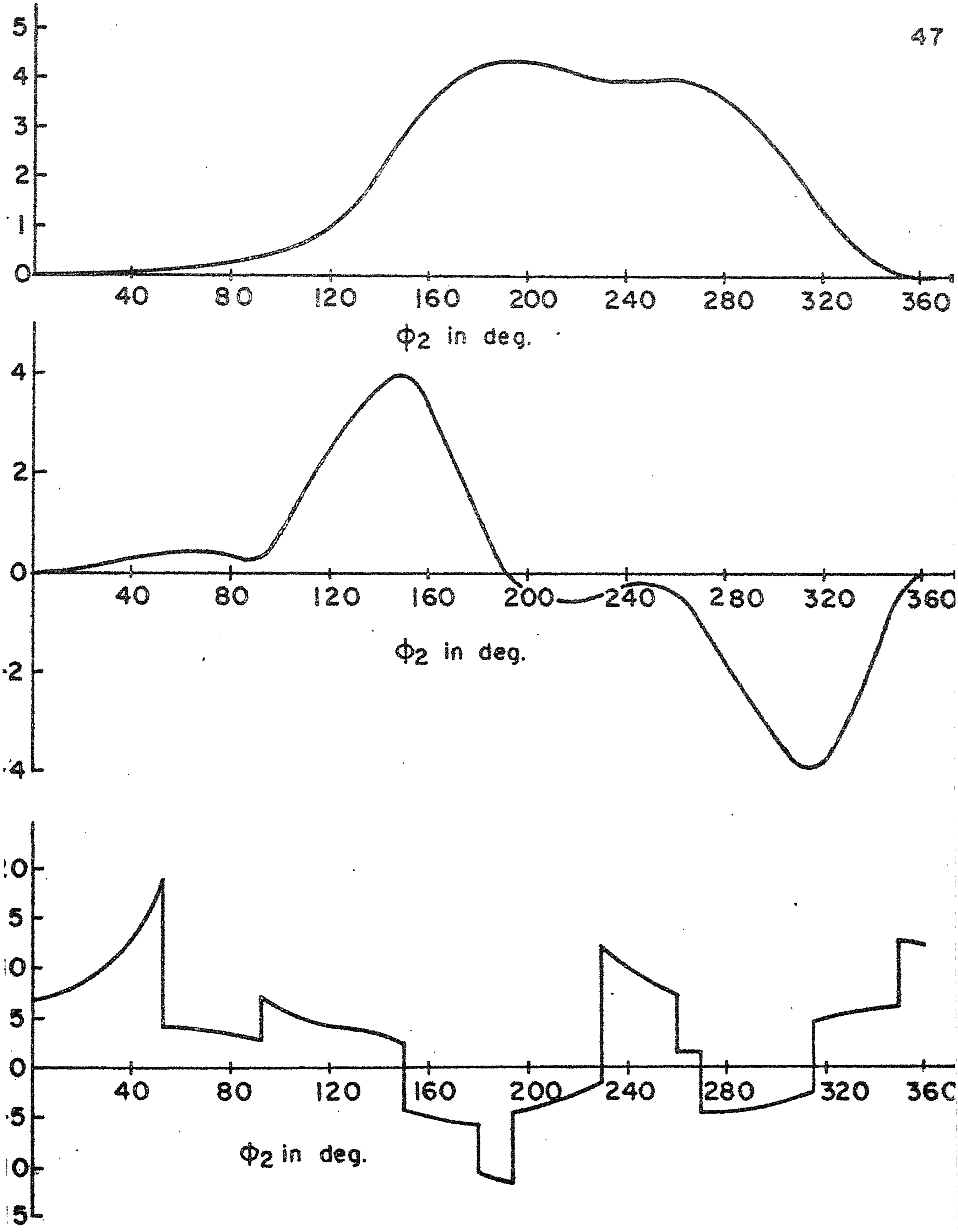


Fig. 4.3a Linear displacement, velocity and acceleration curv

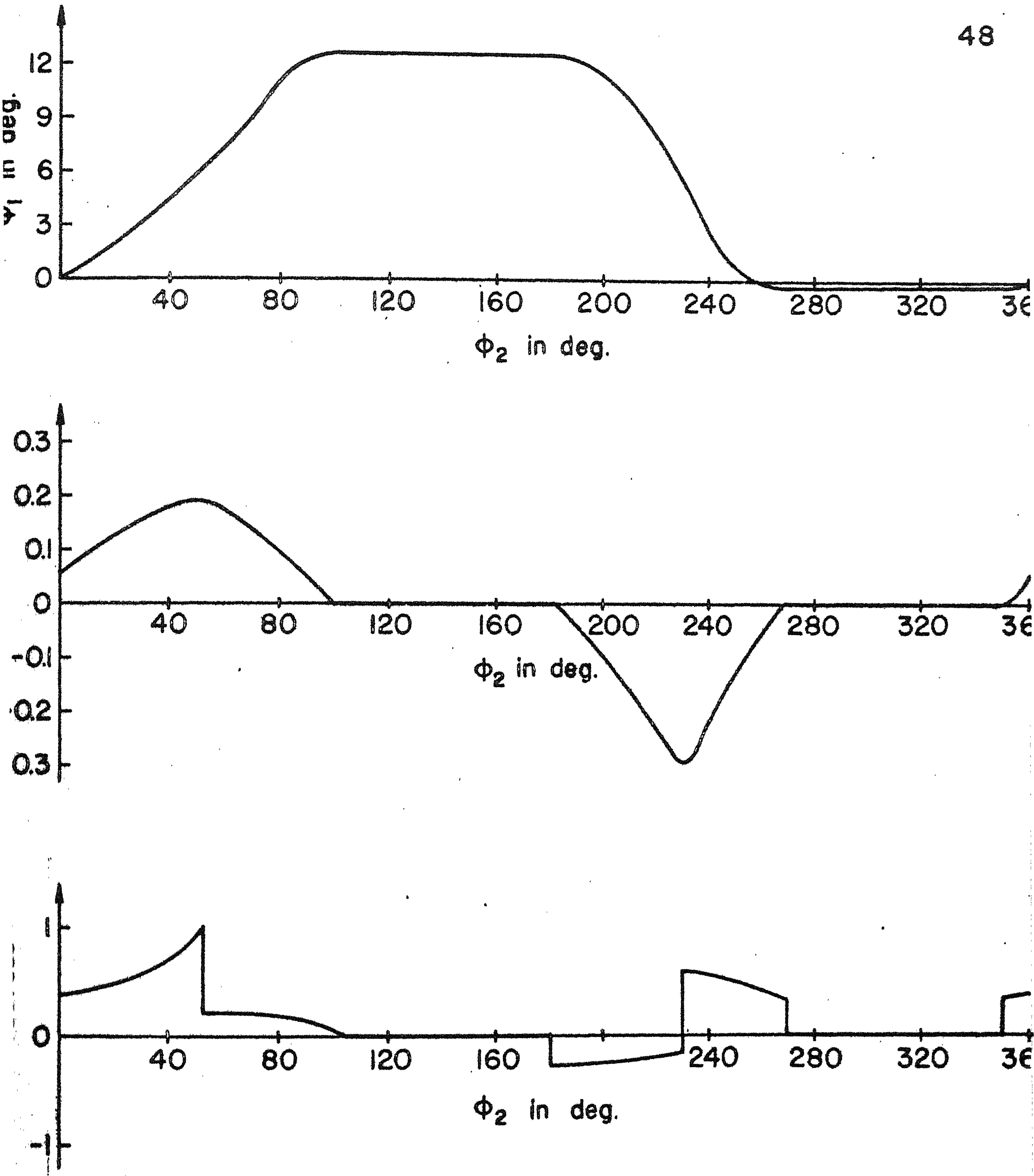


Fig:4.3b Angular displacement, angular velocity and angular acceleration curves of type I cam profile, general cas

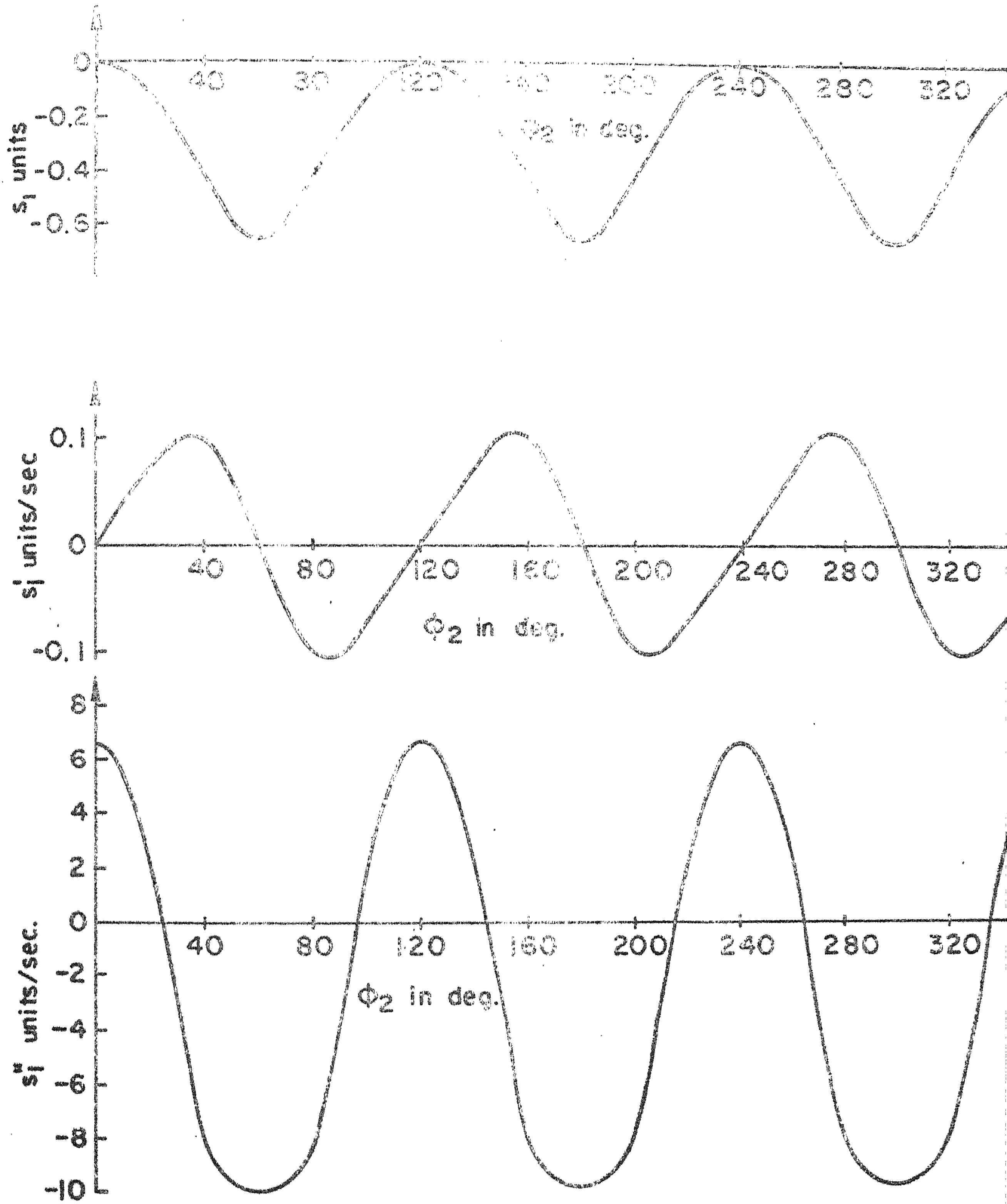


Fig. 4.4a Linear displacement, velocity and accelleratio[n] curves of type II cam profile, General case

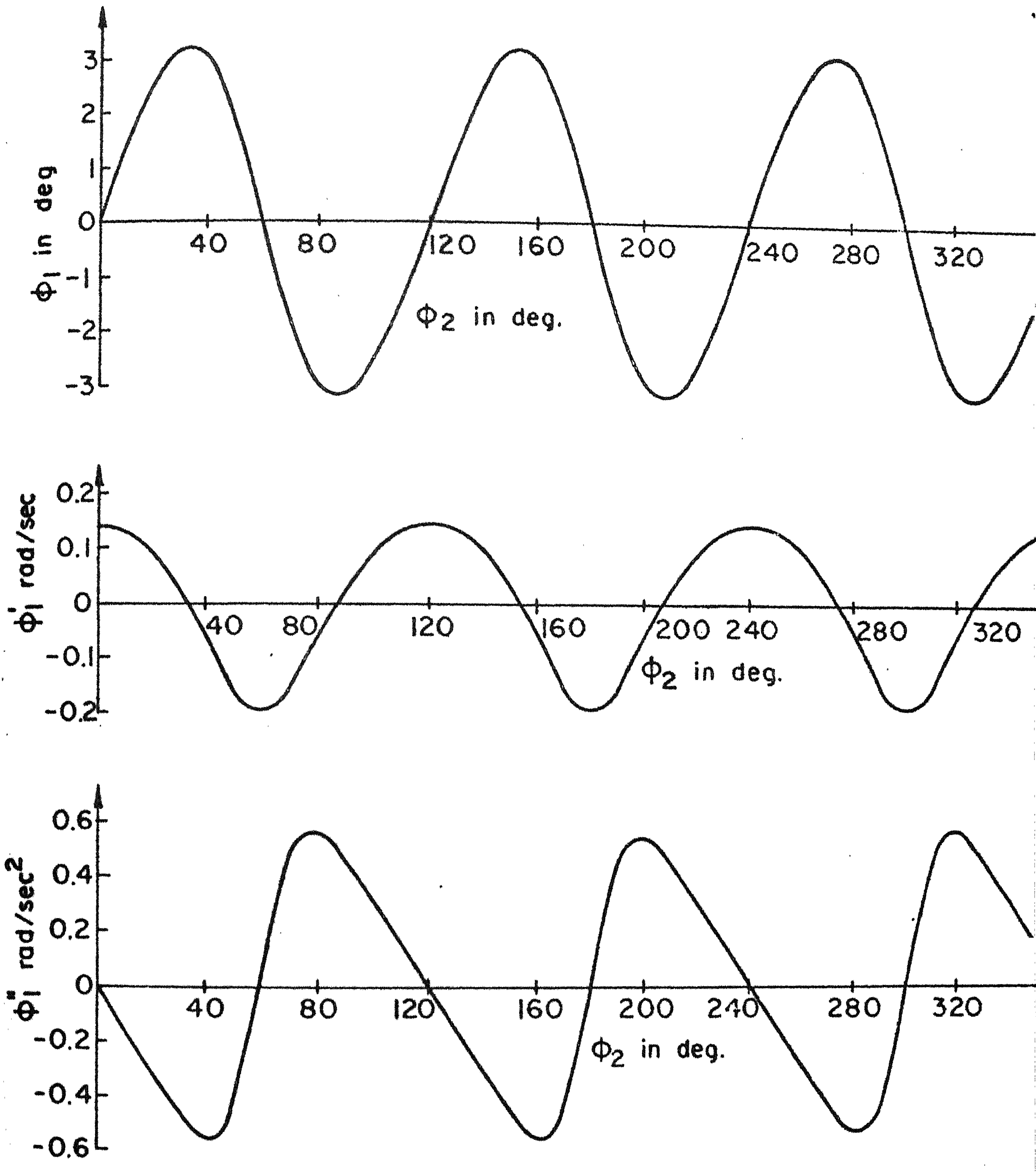


Fig. 4.4b Angular displacement, angular velocity and angu acceleration curves of Type II cam profile, General case.

Chapter 5

CONCLUSIONS AND DISCUSSIONS

5.1 Concluding Remarks

The present work provides an analytical approach to the kinematic analysis of certain types of constant-breadth cam-follower mechanisms. The proposed approach is general in nature and can be used for kinematic analysis of any profile-closed higher pair mechanism. The proposed approach has certain advantages over the graphical approach that was found prevalent in the literature. The analytical approach presented provides closed form parametric equations of the displacement, the velocity and the acceleration of the follower. The main difficulty in implementing an analytical procedure for constant-breadth cam-follower mechanism is that the cam profile cannot be represented by one single parametric equation. In other words each constituent section of a constant-breadth cam profile needs to be described by a different parametric equation. Because of this drawback it is difficult to locate that section of the constant-breadth cam profile in which the follower profile is making a contact at any given instant. Particularly in the case of a translating as well as an oscillating follower case, it is ver essential to select those sections of the profile which are contacting the follower surfaces at every instant of angular rotation during one complete cycle.

In the present work, a search scheme has been developed that helps in the process of locating the point of contact. In some cases the point of contact is evaluated by using a numerical iterative scheme. It is to be noted that inspite of these numerical procedures the final form of the displacement equation isstill parametric in nature.

The objective of the present work was to develop procedures of kinematic analysis. It is, however, felt that the equations developed can be used for any synthesis procedure. For instance, if the limiting values of the acceleration of the follower, the radius of curvature at any point on the cam profile etc. are given, one can use appropriate equations developed in this work to determine the parameters that define the cam profile. The equations developed and the procedure worked out was used in developing a set of computer programmes. The programmes were written in FORTRAN IV and implemented on DEC 1090. These programmes could be used for computer-aided, interactive, graphical design procedures.

5.2 Suggestions for Further Work

The present work was limited to the kinematic analysis of only two cam profiles. However the approach followed here is general in nature and can be extended to the kinematic analysis of several other profile-closed higher pair mechanisms. Examples of such other mechanisms are "Roulettes and Glissettes" [11], the profiles of rotor and stator of a Wankel Engine etc.

The present work does not deal with the aspects of dynamic analysis of constant-breadth cam-follower mechanisms. It is however necessary to carry out such analysis so as to find the optimum value of the operating speed of the cam, the values of cam and follower link masses and the permissible clearances in the follower groove.

Since the profiles of constant-breadth cam-follower mechanisms are geometrically complex in nature, it has been found extremely useful to draw such profiles using the techniques of computer graphics. Besides sketching these profiles, one can also develop an interactive programme using graphical displays for the design of constant-breadth cam-follower mechanisms. Such programme can also display the displacement, the velocity and the acceleration curves.

APPENDIX I

For the case of a Type I constant-breadth cam driving an on-centre translating, flat-faced follower as shown in Figure 2.2a alongwith the set of coordinate systems shown in Figure 2.2b, following are the required transformation matrices. It should be noted that the pivot is at point B, a vertex of the ΔABC.

$$M_{\bar{2}2} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 \\ \sin\phi_2 & \cos\phi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad L_{\bar{2}2} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 \\ \sin\phi_2 & \cos\phi_2 \end{bmatrix},$$

$$M_{\bar{2}1} = \begin{bmatrix} 1 & 0 & d+s_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad L_{\bar{2}1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For the case of a Type I constant-breadth cam driving an offset translating follower as shown in Figure 2.3a alongwith the set of coordinate systems shown in Figure 2.3b, the required transformation matrices are same as above except $M_{\bar{2}1}$, which is given below. It should be noted that the pivot is chosen ℓ and m distances from B in horizontal and vertical directions.

$$M_{\bar{2}1} = \begin{bmatrix} 1 & 0 & d-\ell+s_1 \\ 0 & 1 & -m \\ 0 & 0 & 1 \end{bmatrix}$$

APPENDIX II

For the case of a Type II constant-breadth cam driving an on-centre translating follower as shown in Figure 3.2a alongwith the set of coordinate systems shown in Figure 3.2b, the required transformation matrices $M_{\bar{2}2}$, $L_{\bar{2}2}$ and $L_{\bar{2}1}$ are still the same as in Appendix I. The matrix $M_{\bar{2}1}$ is given below. It should be noted that the pivot is at the point O_f, the centre of the fixed circle.

$$M_{\bar{2}1} = \begin{bmatrix} 1 & 0 & s_o+s_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For the case of a Type II constant-breadth cam driving an oscillating follower as shown in Figure 3.3a alongwith the set of coordinate systems shown in Figure 3.3b, the required transformation matrices $M_{\bar{2}2}$ and $L_{\bar{2}2}$ are still the same as in Appendix I. The matrices $M_{\bar{2}1}$ and $L_{\bar{2}1}$ are given below. The cam pivot is chosen at the point O_f, the fixed circle centre and the pivot of the oscillating follower is taken at a distance H from O_f in horizontal direction

$$M_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & H \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad L_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

APPENDIX III

For the case of a Type I cam profile driving a translating as well as an oscillating follower as shown in Figure 4.1a alongwith the set of coordinate systems shown in Figure 4.1b, the required transformation matrices $M_{\bar{2}2}$ and $L_{\bar{2}2}$ are the same as in Appendix I. The matrices $M_{\bar{2}1}$ and $L_{\bar{2}1}$ are listed below.

$$M_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & s_1\cos\phi_1+h \\ \sin\phi_1 & \cos\phi_1 & s_1\sin\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \quad L_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

The expressions for θ'_Q and θ'_P, θ''_Q and θ''_P used in Equations (4.9) and (4.10), (4.11) and (4.12) are as follows:

$$\theta'_Q = \theta'_P = 2(K_1-K_2)/K_2$$

$$\theta''_Q = \theta''_P = 2\{K_3(K'_1-K'_2)-(K_1-K_2)K'_3\}/K_3^2$$

where

$$K_1 = c_3(c'_1+c'_2)$$

$$K_2 = (c_1+c_2)c'_3$$

$$K_3 = (c_1+c_2)^2+c_3^2$$

$$K'_1 = c'_3(c'_1+c'_2) + c_3(c''_1+c''_2)$$

$$K'_2 = (c'_1+c'_2)c'_3 + (c_1+c_2)c''_3$$

$$K'_3 = 2(c_1+c_2)(c'_1+c'_2) + 2c_3c'_3$$

$$c_1 = -(k_6+h\sin\phi_2)$$

$$c_2 = \{k_6^2+k_4^2+h^2+2h(k_6\sin\phi_2-k_4\cos\phi_2)-(k_5-k)^2\}^{\frac{1}{2}}$$

$$c_3 = k_5+h\cos\phi_2-k-k_4$$

$$c_1' = -h\cos\phi_2$$
$$c_2' = h\{k_6\cos\phi_2 + k_4\sin\phi_2\}/c_2$$
$$c_3' = -h\sin\phi_2$$
$$c_1'' = h\sin\phi_2$$
$$c_2'' = \{h(k_4\cos\phi_2 - k_6\sin\phi_2) - c_2'^2\}/c_2$$
$$c_3'' = -h\cos\phi_2$$

For the case of a Type II cam profile driving a translating as well as an oscillating follower as shown in Figure 4.2a alongwith the set of coordinate systems shown in Figure 4.2b the transformation matrices $M_{\bar{2}2}$, and $L_{\bar{2}2}$ are the same as in Appendix I. The transformation matrices $M_{\bar{2}1}$ and $L_{\bar{2}1}$ are listed below.

$$M_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & H + s_1\cos\phi_1 \\ \sin\phi_1 & \cos\phi_1 & s_1\sin\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \qquad L_{\bar{2}1} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

The Equations (4.20) and (4.21) given in Section 4.2 are

$$\phi_1' = 1 + \{K_1/(K_2 - K_3)\}$$
$$\phi_1'' = \{(K_2 - K_3)K_1' - K_1(K_2' - K_3')\}/(K_2 - K_3)^2$$

where

$$K_1 = \frac{3H}{R}\sin(\phi_2 + \frac{\theta_P}{4})$$
$$K_2 = \frac{3}{4}\sin(\frac{3\theta_P}{4})$$
$$K_3 = \frac{3H}{4R}\sin(\phi_2 + \frac{\theta_P}{4})$$
$$K_1' = \frac{3H}{R}\{\cos(\phi_2 + \frac{\theta_P}{4}) + \frac{\theta_P'}{4}\cos(\phi_2 + \frac{\theta_P}{4})\}$$
$$K_2' = \frac{9\theta_P'}{16}\cos(\frac{3\theta_P}{4})$$
$$K_3' = \frac{3H}{4R}\{\cos(\phi_2 + \frac{\theta_P}{4}) + \frac{\theta_P'}{4}\cos(\phi_2 + \frac{\theta_P}{4})\}$$

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